

GIUH Based Transfer Function for Gomti River Basin of India

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Abstract

Geomorphologic instantaneous unit hydrograph (GIUH) can be used as a transfer function for modeling the transformation of excess rainfall into surface runoff, in which excess rainfall is an excitation (i.e. production function) to the hydrologic system. These models can be used to predict / forecast the temporal variation of the surface runoff at the outlet of ungauged basin, which is useful in the hydrologic / environmental engineering applications. The present study deals with the geomorphometric investigation and provides an efficient solution approach to derive the GIUH based transfer function and thus geomorphologic unit hydrograph (GUH) for the basin. Since, Gomti river basin is ungauged, therefore, to test the effectiveness of the approach two cases were considered. Firstly, the approach was tested on the catchment for which published UH data was available; and secondly, the approach was applied for the Gomti river basin for the derivation of GUH. To verify the derived GUH of the Gomti basin, a comparison was performed with the synthetic unit hydrograph (SUH) obtained from the Central Water Commission (CWC) procedure. Based on the comparison of the result, it may be revealed that the GUH with dynamic flow velocity of 0.68 m/s was close to the SUH.

Key words: Basin, CWC, Direct runoff; Geomorphology; Gomti river; Geomorphologic instantaneous unit hydrograph; Transfer function; Geomorphologic unit hydrograph, SRTM, Synthetic unit hydrograph, Ungauged basin

Introduction

Water resources planning, development and operation of various schemes requires accurate estimation of hydrologic response of the basin. In surface hydrology, rainfall-runoff and soil erosion process are very important and needs good understanding. These hydrological processes are nonlinear and involve various climatic, topographic, soils, land use information. To develop a good understanding and hydrological model for these processes, a reliable and wide variety geophysical and hydro-meteorological data are required. These models may be broadly classified as empirical, conceptual, lumped, and physically based distributed models. Empirical models have their own limitations as they are site specific, whereas, conceptual models are flexible and based on the simplification/approximation of physical concepts of the processes. Therefore, conceptual model has a room for theoretical simulation of individual components of the process. For example, Nash based instantaneous unit hydrograph (IUH)

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model (Nash, 1957) is based on the concept of 'a cascade of linear reservoirs' in the watershed. The physically based distributed models on the other hand, need rigorous computational skill for solving the governing equations and therefore, require a significant quantum of watershed information along with event based rainfall and runoff data for the calibration of model parameters (Singh and Frevert, 2002 a, b).

It is the fact that most of the Indian as well the watersheds of developing countries do not have sufficient historical hydrological records and detailed watershed information needed for physically based distributed models. Therefore, a simplified representation of the rainfall-runoff process through the input-output linkage without a detailed physical description of the process can be used and this is the basis of the linear theory of hydrologic systems. For mathematical description of linear hydrologic system, the unit hydrograph (UH) or unit pulse response function can be employed for system identification in hydrologic analysis (Sherman, 1932; Dooge, 1959, 1973; Chow et al., 1988; Singh, 1988a). Therefore, geomorphology based approach becomes a one of the most popular modeling tools for the computation of runoff hydrographs under these circumstances (Rodriguez-Iturbe and Valdes, 1979; Rodriguez-Iturbe et al., 1982; Yen and Lee, 1997; etc.).

Geomorphology of a river basin describes the status of topographic features of the surfaces and streams, and its relationship with hydrology provides the geomorphological control on basin hydrology (Jain and Sinha, 2003). Geomorphology reflects the topographic and geometric properties of the watershed and its drainage channel network. It controls the hydrologic processes from rainfall to runoff, and the subsequent flow routing through the drainage network. The role of basin geomorphology in controlling the hydrological response of a river basin is known for a long time. Moreover, for any infrastructural development, it is very useful tool for first hand overview of the basin. It is advantageous in case of laying out the urban drainage and irrigation canal system, aqueducts, study the physiographic impacts on environment, and selection of silt disposal site, hydropower site (Sarkar and Gundekar, 2007), recharge zone, percolation tank, retention tank, dam site, etc. This drainage network of the river basin can provide a significant contribution towards flood management and water logging program (Jain and Sinha, 2003). Earlier works have provided an understanding of basin geomorphology-hydrology relationship through empirical relationships (Snyder, 1938; Horton, 1945; Taylor and Schwartz, 1952; Singh, 1988b; Singh, 1989; Jain and Sinha, 2003; etc.). Snyder (1938) proposed synthetic unit hydrograph approach (SUH) for ungauged basin as a function of catchment area, basin shape, topography, channel slope, stream density and channel storage; and derived the basin coefficient by averaging out other parameters.

Furthermore, the hydrologic response is a function of climatic parameters, landuse, soil parameters and topography and therefore, for any physical based models requires time to time change in their parameters due to variation encountered with respect to the gradual climatic changes and landuse of the watersheds (Rodriguez-Iturbe et al., 1982). However, the geomorphological parameters are mostly time-invariant in nature and therefore, geomorphology based approach could be the most suitable technique for modeling the rainfall-runoff process for ungauged catchments. The rainfall pattern, in general is undergoing a change due to global changes in atmospheric conditions. Further, because of different activities in the watershed, its land use is also having a gradual changes and this has an impact on the characteristics of the runoff produced from the watersheds. Thus, the geomorphological instantaneous unit hydrograph (GIUH) (Rodriguez-Iturbe and Valdes, 1979) and further simplified by Gupta et al. (1980) is a hydrological model that relates the geomorphological features of a basin to its response to rainfall. They can be applied to ungauged basins having scarce hydrologic data (Al-Wagdany et al., 1998). The derivation of

the GIUH uses the assumption that a stream of a certain order has a known linear response function of the familiar or complex probability distributions (Rodriguez-Iturbe and Valdes, 1979; Kirshen and Bras, 1983; Rinaldo et al., 1991; Jin, 1992; Fleurant et al., 2006; etc.). The effect of linear channels in the hydrologic response was introduced by Kirshen and Bras (1983). Thus, the GIUH based transfer function approach is applicable in such a situation where rainfall data is available but runoff data are not, and it is a more powerful technique for the flood estimation than the commonly used parametric Clark model (Clark, 1945) and Nash's cascade technique (Nash, 1957) (Yen and Lee, 1997; Bhaskar et al., 1997; Jain et al., 2000; Lohani et al., 2001; Kumar et al., 2002; Sarangi et al., 2007; Bhadra et al., 2008; etc.).

Another advantage of GIUH technique is its potential for deriving the unit hydrograph (UH) using the geomorphologic characteristics obtainable from topographic map / remote sensing, possibly linked with geographic information system (GIS) and digital elevation model (DEM) (Rodriguez-Iturbe and Valdes, 1979; Rosso, 1984; Sahoo et al., 2006; Kumar et al., 2007; etc.). However, the GIUH technique is applicable for the estimation of the direct runoff component of the stream flow and hence, can be used to generate the direct runoff hydrograph (DRH). Once the DRH is computed, the flood hydrograph can be simply obtained by adding the base flow component.

Therefore, based on the foregoing discussions, and the interrelationship between the geomorphology and hydrology, the present study was carried out with the objectives: (i) to present an efficient solution approach for generating geomorphologic transfer function i.e. GIUH; (ii) to test the applicability of proposed approach on the published data of Bhunya et al. (2008) (iii) to investigate the geomorphology of the Gomti river basin and estimate the dimensionless Horton's ratios; and (iv) to derive the geomorphologic unit hydrograph (i.e. GUH) for Gomti river basin using the Horton's ratios.

Geomorphologic Parameters

Certain characteristics of the drainage basins reflect hydrologic behavior and are therefore, useful, when quantified, in evaluating the hydrologic response of the basins. These characteristics relate to the physical characteristics of the drainage basin as well as of the drainage network. Physical characteristics of the drainage basin include drainage area, basin shape, ground slope, and centroid (i.e. centre of gravity of the basin). Channel characteristics include channel order, channel length, channel slope, channel profile, and drainage density.

Basin order and channel order

Drainage areas may be characterized in terms of the hierarchy of stream ordering. The order of the basin is the order of its highest-order channel. It can be stated that the uppermost channel receives water from overland surface and direct towards the outlet through the drainage network. This upper most channel joins another channel and form the higher order channel, and so on. The first order channel is defined as those channels that receive water entirely from overland surface and does not have tributaries. The junction of two first order streams forms the second order stream. It means the higher order drain carries more water than the lower order drains. A second order channel receives flow from the two first order stream. When the two second order streams joins together forms the third order stream, and so on. This scheme of stream ordering is referred to as the Horton-Strahler ordering scheme (Horton, 1945; Strahler, 1957). A watershed is described as first-, second-, third-, or higher order, depending upon the stream order at the outlet.

Basin area

Basin area is defined as the area contained within the vertical projection of the drainage divide on a horizontal plane. Some areas in the drainage basin do not contribute to the runoff and are termed as closed drainage. These area may be lakes, swamps etc.

Basin shape

The basin shape may influence the hydrograph shape, especially for small watersheds. For example, if the watershed is long and narrow, then it will take longer time for water to travel from the most extreme point to the outlet and the resulting hydrograph shape is flatter. For more compacted watershed, the runoff hydrograph is expected to be sharper with a greater peak and shorter duration. Numerous symmetrical and irregular forms of drainage areas are encountered in practice. To define the basin shape, a multitude of dimensionless parameters were used to quantify and these are: form factor, shape factor, elongation ratio, circulatory ratio, and compactness coefficient (Table 1). These factors involve watershed length, area and perimeter. The watershed length can be defined as the length of the main stream from its source (projected to the perimeter) to the outlet. Clearly, the form factor is less than unity and its reciprocal, the shape factor, is greater than unity. A square drainage basin has the shape factor greater than unity. The elongation ratio, circulatory ratio, and compactness coefficient approach to unity as the watershed shape approached to circle.

Parameter	Definition	Formula	Value
(Authors)		2	
Form factor	Watershed area	$= A/L^2$	< 1
	$\overline{(Watershed length)^2}$		
Shape factor, B_s	$(Watershed length)^2$	$=L^2/A$	> 1
	Watershed area		
Elongation ratio	Diameter of circle of watershed area	$1.128 A^{0.5}$	≤1
	Watershed length	$=$ $\frac{1}{L}$	
Circulatory ratio	Watershed area	$-\frac{12.57 A}{12.57 A}$	≤1
	Area of circle of watershed area	$-P_r^2$	
Compactness	Watershed perimeter	$-0.2821P_{r}$	≥1
coefficient	Perimeter of circle of watershed area	$-A^{0.5}$	

Table 1 Watershed shape parameters (Singh, 1988)

A = watershed area; L = watershed length; P_r = watershed perimeter

Basin slope

Basin slope has a pronounced effect on the velocity of overland flow, watershed erosion potential, and local wind systems. Average basin slope is defined as (Singh, 1989)

$$S = h/L$$

(1)

where S is the average basin slope (m/m), h is the fall (m) (i.e. difference in maximum and minimum elevations), and L is the horizontal distance (m) over which the fall occurs.

Drainage basin similarity

Strahler (1957) hypothesized the drainage basin similarity as follows:

$$A/L_b^2 = C$$

where, *A* is the area of the drainage basin $[L^2]$, L_b is the length of the basin [L] and *C* is the constant for basin similarity. The value of C will be nearly equal for geomorphologically similar basins.

Drainage density

Drainage density is defined as the length of drainage per unit area. This term was first introduced by Horton (1932) and is expressed as

$$D_d = L/A \tag{3}$$

where, D_d is the drainage density, *L* is the total length of the drains in the basin and A is the area of the drainage basin.

Eq. (3) can be further expanded as follows:

$$D_{d} = \frac{\sum_{i=1}^{\Omega} \sum_{j=1}^{N_{i}} L_{i,j}}{A}$$
(4)

where, Ω is the highest order of the stream, N_i is the number of i – th order drain and $L_{i,j}$ is the length of j – th stream of i – th order drain. Above relationships can be simplified as follows:

$$D_d = \frac{\overline{L}_1 R_b^{\Omega - 1}}{A} \cdot \frac{r^{\Omega} - 1}{r - 1} \tag{5}$$

where, \overline{L}_{l} is the average length of the first order drain (= $\sum_{j=1}^{N_{l}} L_{l,j} / N_{l}$) and ratio *r* is defined as follows:

$$r = R_l / R_b \tag{6}$$

where R_l and R_b are the stream-length, and bifurcation ratio, respectively.

Determination of Horton's ratio

Three of Horton's ratios namely bifurcation ratio (R_b), stream-length ratio (R_l) and stream-area ratio (R_a) are unique representative parameters for a given watershed and are fixed values for a given watershed system.

Bifurcation ratio R_b

The number of channels of a given order in a drainage basin is a function of the nature of the surface of that drainage basin. In general, the greater the infiltration of the soil material covering the basin, the fewer will be the number of channels required to carry the remaining runoff water. Moreover, larger the number of channels of a given order, the smaller is the area drained by each channel order. A dimensionless parameter based on the number of channels with respect to their order is termed as bifurcation ratio and is useful in defining the watershed response. The bifurcation ratio is given as follows.

$$R_b = N_i / N_{i+1}$$

(7)

(2)

where R_b is the bifurcation ratio, N_i and N_{i+1} are the number of streams in order *i* and *i*+1 respectively, $i = 1, 2, ..., \Omega$ and Ω is the highest stream order of the watershed. The value of R_b for watersheds varies between 3 to 5. This law is an expression of topological phenomenon, and is a measure of drainage efficiency.

Stream-length ratio R₁

This refers to length of channels of each order. The average length of channels of each higher order increase as a geometric sequence, which can be further explained as: the first order channels are the shortest of all the channels and the length increase geometrically as the order increases. This relation is called Horton's law of channel length and can be formulated as follows.

$$\overline{L}_{i} = \overline{L}_{1} R_{l}^{i-1}$$
(8)

where \overline{L}_i is the average length of channel of order *i*, R_i is the stream-length ratio. The R_i is mathematically expressed as follows.

$$R_{l} = \overline{L}_{i+1} / \overline{L}_{i}$$
(9)
where,

$$\overline{L}_{i} = \frac{1}{N_{i}} \sum_{j=1}^{N_{i}} L_{i,j}$$
(10)

The lengths of channels of a given order are determined largely by the type of soil covering the drainage basin. Generally, more pervious the soil, longer will be the channel length of a given order. Also, higher is the R_i more will be the imperviousness. Generally it varies between 1.5 to 3.5.

Stream-area ratio R_a

The Channel area of order *i*, A_i is the area of the watershed that contributes to the channel segment of order *i* and all lower order channels. It can be quantified as:

$$\overline{A}_i = \overline{A}_i R_a^{i-1} \tag{11}$$

where \overline{A}_i is the average area of order *i* and R_a is the stream area ratio, and are given as follows:

$$R_{a} = A_{i+1} / A_{i}$$
(12)
$$\overline{A}_{i} = \frac{1}{N_{i}} \sum_{j=1}^{N_{i}} A_{i,j}$$
(13)

where $A_{i,i}$ is the total area that drains into the j^{th} stream of order *i*.

Derivation of Geomorphologic Unit Hydrograph (GUH)

This section describes the potential of GIUH for deriving the SUH, and solution procedure adopted to solve the Nash based GIUH and GUH.

Background of GIUH based approach

The GIUH theory was introduced by Rodriguez-Iturbe and Valdes (1979) by relating the peak discharge and time to peak discharge with the geomorphologic characteristics of the catchment and a dynamic

velocity parameter. This pioneering work of Rodriguez-Iturbe and Valdes (1979), which explicitly integrate the geomorphologic details and the climatologic characteristics of the basin, in a framework of travel time distribution, is a boon for stream flow synthesis in the basin having no or scanty information of flow data. This formulation of GIUH is based on the probability density function (*pdf*) of the time history of a randomly chosen drop of effective rainfall arrived to the trapping state of a hypothetical basin, treated as a continuous Markovian process, where the state is the order of the stream in which the drop is located at any time. The value at the mode of this *pdf* produces the main characteristics of GIUH. Rodriguez-Iturbe and Valdes (1979) derived the peak and time to peak characteristics of the IUH as function of Horton's order ratios (Horton, 1945), and are expressed as follows (Kumar et al., 2007).

$$q_p = 1.31 R_l^{0.43} V / L_{\Omega}$$
(14)

$$t_p = 0.44 \left(L_{\Omega} / V \right) \left(R_b / R_a \right)^{0.55} \cdot R_l^{-0.38}$$
(15)

where q_p is the peak flow (h⁻¹), t_p is the time to peak (h), L_{Ω} is the length of the highest order stream (km), V is the dynamic velocity parameter (m s⁻¹). Multiplying eqs. (14) and (15) a non-dimensional term $q_p * t_p$ is derived at.

$$q_{p} * t_{p} = 0.5764 [R_{b} / R_{a}]^{0.55} R_{l}^{0.05}$$
(16)

The term $q_p * t_p$ is not dependent on the velocity and thereby on the storm characteristics and hence, it is a function of only the geomorphologic characteristics of the basin. The dynamic velocity parameter in the formulation of GIUH incorporates the effect of climatic variation. Rodriguez-Iturbe et al. (1979) showed the dynamic velocity parameter of the GIUH can be taken as the velocity at the peak discharge time for a given rainfall-runoff event in the catchment. Valdes et al. (1979) compared the GIUHs for some real world basins with the IUHs derived from the discharge hydrograph produced by a physically based rainfallrunoff model of the same basins and found them to be remarkably similar. For few of Indian catchments this GIUH based approach was successfully demonstrated (Kumar et al., 2002; 2007). In the present study, the Nash based GIUH model was developed for the Gomti river basin.

The Nash model

The Nash model (Nash, 1957) is based on the concept of routing of the instantaneous inflow through a cascade of linear reservoirs with equal storage coefficient. The Nash model can be expressed as follows:

$$u(t) = \frac{1}{k \Gamma(n)} (t/k)^{n-1} \exp(-t/k)$$
(17)

where u(t) is the ordinates of IUH (hour⁻¹), *t* is the sampling time interval (hour), *n* and *k* are the parameters of the Nash model, in which *n* is the number of linear reservoirs, and *k* is the storage coefficient (hour).

A unit hydrograph (UH) of desired duration (D) may be derived by using the following expression:

$$U(D,t) = \frac{1}{D} [I(n,t/k) - I(n,(t-D)/k)]$$
(18)

where U(D, t) denotes the ordinates of *D* hour UH (hour⁻¹), *t* is the sampling time interval (hour), I(n, t/k) is the incomplete gamma function of order *n* at (t/k), and *D* is the duration of UH (hour).

Geomorphology based parameter estimation of Nash's GIUH

The complete shape of the GIUH can be obtained by linking the q_p and t_p of the GIUH with scale (*k*) and shape (*n*) parameters of the Nash model. By equating the first derivative with respect to *t* of eq (17) to zero, *t* becomes the time to peak discharge, t_p . Thus, taking the natural logarithm of both sides of eq. (17), differentiating with respect to *t* and by simplification eq. (19) is derived.

$$\partial_{1} \left[(n-1) \right]$$

$$\frac{\partial}{\partial t}\ln[u(t)] = \left\lfloor -\frac{1}{k} + \frac{(n-1)}{t} \right\rfloor$$
(19)

Equating eq. (19) to zero results in by replacing t with t_p ,

$$t = t_p = k(n-1) \tag{20}$$

Simplifying the value of t_p from eq. (20) in eq. (17) and simplifying yields

$$q_{p} = \frac{1}{k\Gamma(n)} \exp[-(n-1)] \cdot (n-1)^{n-1}$$
(21)

Combining eqs. (20) and (21) results:

$$q_{p} \cdot t_{p} = \frac{(n-1)}{\Gamma(n)} \exp[-(n-1)] \cdot (n-1)^{n-1}$$
(22)

Equating eq. (22) with eq. (16) results in:

$$\frac{(n-1)}{\Gamma(n)} \cdot \exp[-(n-1)] \cdot (n-1)^{n-1} = 0.5764 [R_b / R_a]^{0.55} \times R_l^{0.05}$$
(23)

The Nash parameter n, can be obtained by solving eq. (23) using the Newton-Rapson method. Procedural step of estimation of n using Newton-Rapson method is given in appendix-I. The Nash's parameter k for the given velocity V is obtained using eqs. (15) and (20) and the known value of the parameter n as follows:

$$k = \frac{0.44 L_{\Omega}}{V} \cdot \left[\frac{R_b}{R_a}\right]^{0.55} \cdot R_l^{-0.38} \cdot \frac{1}{(n-1)}$$
(24)

The derived values of n and k are used to determine the complete shape of the Nash based GIUH using eq. (17). Subsequently, the D-hour UH can be derived from eq. (18). The direct runoff hydrograph (DRH) is estimated by convoluting the excess rainfall hydrograph with the UH.

Application of the Model and Results

Gomti basin is ungauged basin; therefore, testing the accuracy of derived GUH for this basin using the aforesaid approach is quite difficult. Thus, for systematic application of the proposed approach, two cases were considered: (i) Case I: Application of the model on published data of Burhner catchment taken from Bhunya et al. (2008); and (ii) Case II: Application of the model for Gomti river basin.

Case I: Application of the model on published data of Burhner catchment

India is divided into seven hydro-meteorological zones and is further divided into twenty six hydrometeorological sub-zones [i.e. sub-zones 1(a) to 1(g), 2(a) to 2(c), 3(a) to 3(i), 4(a) to 4(c), 5(a), 5(b), 6 and 7] (Jain et al., 2007). The Burhner catchment was picked up from hydro-meteorological subzone 3(c) which comprises of Upper Narmada and Tapi basins. It lies between 80°36' and 81°23' E longitudes and 22°00' and 22°56' N latitude. It has a continental type of climate with hot summers and cold winters and receives most rainfall from the south-west (SW) monsoon during the months of June to October. Mean annual rainfall varies approximately from 800 to 1600 mm. The main soil group is black soil, comprising

different types, namely, deep black, medium black and shallow black soil. It is sixth order catchment comprised of about 4103 km² area. Length of maximum order stream is 138 km, whereas R_a , R_b , R_l are 3.96, 3.523, 1.787 respectively. Peak discharge, Q_p of hydrograph is found to be 69.02 m³/s, whereas, time to peak t_p is of the order of 11 hour and velocity is 4.15 m/s.

Using the aforementioned geomorphologic information, the GUH was derived for the Burhner catchment and compared with the observed UH reported in the study by Bhunya et al. (2008). The comparison of the GUH and observed UH is depicted in Fig. 1. The comparative result of UH for the catchment indicates the suitability of the technique for the derivation of UH and thus DRH.



Fig. 1 Comparison of derived GUH and observed UH of Burhner catchment (Case I)

Case II: Application of the model for Gomti river basin

The river Gomti is one of the major tributary of river Ganga originated from the Pilibhit district of the Uttar Pradesh and joins the river Ganges at Varanasi after traveling approximately 662.5 km. It drains approximately 30407.2 km² area of the central-east part of the Uttar Pradesh. Gomti river basin extended between the latitude of 25°23'12.62" to 28°46'58.75" N and longitude of 79°57' 33.76" to 83°11'13.25" E and belongs to hydro-meteorological subzone 1(f). The digital elevation model (DEM) and the extracted drainage network of the basin using the ArcGIS are shown in Fig. 2(a) and 2(b), respectively. The shape of the basin is nearly oblong in nature. Topography is nearly undulating with high terrains at upstream end of the basin. The maximum at upstream end and minimum elevation at downstream end of the basin are 227 m and 58 m above msl, respectively. The climate of the basin is ranging from semi arid to sub humid tropical with average annual rainfall at different locations is 850-1100 mm. Approximately, 75 percent of total rainfall is due to the occurrence of North-West Monsoon (Mid June to Mid October). The mean minimum and maximum temperature over the basin is 4.5° to 46.0° C with daily mean sunshine of 8 hours. The relative humidity varies between 10-90 percent. The potential evapotranspiration experienced in the basin is nearly 1300 mm. Major soil group experienced over the basin is sandy loam to clay loam, however, large variation can be observed throughout the basin. The land use in the basin can be broadly categorized as: agriculture, non-agriculture, pasture fellow, forest, etc. Major crops cultivated in the basin are paddy, wheat, sugarcane, potato, and other vegetable crops depending on the irrigation facilities.



Fig. 2 Digital Elevation Model (DEM) and extracted drainage network of the Gomti river basin

To extract the geomorphologic features of the basin, SRTM (Shuttle Radar Topography Misson) data of 90 x 90 m resolution was used. Based on the study by Haase and Frotscher (2005) it can be stated that the SRTM data sets are of major relevance for providing terrain information in large and transboundary river basins for handling the regional environmental problems, and could be applied in meso / macroscale river network and terrain analyses. Besides these, the application of the SRTM data is cost-efficient and informative and is web-based world wide available and appropriately implemented into a geographic information system (GIS)-framework. The DEM of Gomti river basin (Fig. 2a) from SRTM data set was processed in the ArcGIS 9.1 platform to delineate the basin boundaries and drainage networks (Fig. 2b). The stream ordering was carried out using the DEM of the basin using Strahler's scheme followed by establishing the flow direction, flow accumulation etc. The extracted stream network of different orders is shown in Fig. 3(a), and the highest order of the basin was recorded as 6. Maximum length of the river is found to be 662.5 km. As stated earlier, the aerial contribution to the runoff by different order is also important to understand the runoff and erosion process, and therefore, it needs to be extracted and shown in Fig. 3(b). The number of streams of different orders, length, and corresponding area are presented in Table 2, whereas, the extracted geomorphologic parameters such as drainage area, perimeter of the basin, length of the basin, maximum and minimum elevation, watershed relief, relief ratio, elongation ratio, mean slope, drainage density, stream frequency, circulatory ratio, farm factor, Horton's bifurcation ratio (Fig. 4), length ratio (Fig. 5), stream-area ratio (Fig. 6), etc. are summarized in Table 3.



Fig. 3 Streams of different orders and corresponding aerial coverage for Gomti river basin



Fig. 4 Estimation of bifurcation ratio (R_b) of the basin



Fig. 5 Estimation of stream length ratio (R_i) of the basin



Fig. 6 Estimation of stream area ratio (R_a) of the basin

omti basin			
Order	Number	Length (km)	Area (km²)
1	1316	4422.90	18577.16
2	291	2243.21	23670.74
3	65	1222.95	28525.32
4	12	797.90	29549.77
5	4	645.34	30252.09
6	1	63.82	30407.20

Table 2 Number,	length,	and	area f	or	streams	of	various	orders	of
Gomti basin									

Parameters	Value		
Area (km ²)	30407.2		
Perimeter (km)	1639.4		
Length of Basin (km)	481.0		
Maximum elevation (m)	227.0		
Minimum elevation (m)	58.0		
Relief (km)	0.169		
Mean slope (km/km)	0.00027		
Stream Characteristics	Numbers	Mean Length (km)	Mean Area (km²)
1 st order stream	1316	3.36	14.12
2 nd order stream	291	7.71	81.34
3 rd order stream	65	18.81	438.85
4 th order stream	12	66.49	2462.48
5 th order stream	4	161.34	7563.02
6 th order stream	1	63.82	30407.20
Ratios			
Bifurcation ratio (R _b)	4.283		
Length ratio (R _I)	2.218		
Stream-area ratio (R _a)	4.772		
Horton ratio (r)	0.518		
Drainage density (D _d)	0.304		
Drainage frequency (F_s)	0.055		
Elongation ratio (Re)	0.144		
Circulatory ratio (Rc)	0.145		
Farm factor (Ff)	0.121		
Shape factor (Fs)	8.24		
Compactness factor (Fc)	2.63		

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Using the aforesaid approach of Nash based GIUH and estimated Horton's ratios (i.e. R_b , R_h , and R_a) the parameter *n* of the Nash model was optimized using the Newton Rapson method (i.e. eqs. 25 through 31). The optimized value of *n* was found to be 3.17. The parameter *k* of the Nash model was estimated for the derived Horton's dimensionless parameters (i.e. R_b , R_h and R_a) at different dynamic velocity of flows using eq. (24). The dynamic velocity of flow can be computed using the approach suggested by Kumar et al. (2002). The estimated values of *n* and *k* for various flow velocities are given in Table 4. Using the estimated values of the *n* and *k* at different velocity of flows *V*, the ordinates of instantaneous unit hydrograph (IUH) was computed using eq. (17), and finally the 1-hour UHs were derived for different velocity of flow using the relationship given by eq. (18). The 1 h-UH at different velocities are depicted in Fig. 7.



Fig. 7 GUH at different velocities and observed UH (CWC procedure) for the Gomti river basin

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V (m/s)	п	<i>k</i> (hour)	V (m/s)	п	k (hour)
0.50	3.1665	18.0463	3.00	3.1665	3.0077
1.00	3.1665	9.0232	3.50	3.1665	2.5780
1.50	3.1665	6.0154	4.00	3.1665	2.2558
2.00	3.1665	4.5116	4.50	3.1665	2.0051
2.50	3.1665	3.6093	5.00	3.1665	1.8046

Table 4 Estimated values of Nash parameters *n* and *k* at different velocity, *V*

To validate the derived GUH for the Gomti river basin, a comparison was presented with the UH computed from the procedure adopted by CWC. The CWC method is well accepted by field engineers / hydrologist for the derivation of UH on the ungauged catchment in India for the estimation of design flood to install the waterway of bridges/cross drainage structures across small and medium streams. In this method a set of equations for UH parameters (viz., peak flow rate, time to peak flow rate and time base of the hydrograph) as a function of slope, length of the longest stream and drainage area were developed for the regions based on the regional analysis of the observed data of 42 representative sub-watersheds (40 sub-watersheds data were collected by Northern and North-eastern Railways, India and 2 sub-watersheds data were collected by Ministry of Transport, India). Computational step to derive the SUH using CWC technique is given in Appendix-II. The computed 1- hour SUH using the CWC procedure is shown in Fig. 7.

It can be revealed from Fig. 7 that the derived GUH with flow velocity of 0.68 m/s (approx) is close to the SUH computed by CWC procedure. Since, CWC technique is independent of the most important variable of the basin i.e. climatic parameter (i.e. dynamic flow velocity) and does not account for the geomorphologic parameters such as characteristics of the drainage network, contributing area of the different order drains and their lengths. Thus, the error associated in the computation of flood hydrograph may be more. Besides this, CWC technique does not give the actual shape of the hydrograph. Hence, the GUH based approach becomes more realistic which provide the complete shape of the UH at different

values of dynamic velocity of flow. Once the dynamic flow velocity is estimated from field visit, the actual UH of the basin can be easily obtained. The average dynamic flow velocity in the Gomti river varies between 0.65 to 0.75 m/s. The DRH and thus the flood hydrographs for the river Gomti can be easily obtained by convoluting the rainfall excess (mm) with ordinates of GUH.

Using the results obtained from the analysis, a relationship between the peak flow rate, dynamic flow velocity, and time to peak flow have been sought for Gomti river basin and are expressed as follows.

 $\begin{array}{ll} Q_p = 0.0289 \times V^{0.9982} & (R^2 = 0.999) & (25) \\ t_p = 19.834 \times V^{1.043} & (R^2 = 0.993) & (26) \\ t_p = 0.488 \times Q_p^{-1.0452} & (R^2 = 0.993) & (27) \\ \text{where, } Q_p \text{ is the peak flow rate (m/s/mm), } V \text{ is the dynamic velocity of flows (m/s), and } t_p \text{ is the time to} \\ \text{peak (hours). Based on the velocities, the peak flow and time to peak can be directly obtained. Equations} \\ (24) \text{ through (27) can be used as a ready reference to the field engineers for design of hydraulic structures.} \end{array}$

Conclusions

This study comprised of geomorphological investigation and development of GUH for Gomti river basin of India. Based on the geomorphological analysis, Gomti river basin is of sixth order and almost flat terrain except at the upstream end of the basin. A detailed feature useful to engineering and environmental studies / application for the basin is summarized in tabular form (Tables 2 and 3). Also, the extracted drainage network of the basin has wide range of application such as installation of the artificial drainage system and laying down the canal system, site identification for rain water harvesting, groundwater recharge and waste disposal, etc. However, the main theme of the paper is to develop a GUH for the Gomti river basin for the estimation of flood hydrograph. To test the accuracy of the GIUH technique, Case-I was considered to derive the GUH for the Burhner catchment of India. Based on the comparison of the derived GUH and observed UH, it may be concluded that the proposed approach is suitable and efficient for the derivation of UH.

On the other hand, Case II presents the application of GIUH technique for the derivation of GUH for Gomti river basin. Three of Horton's ratios of the Gomti river basin were coupled with the parameters of the Nash's IUH model and the GUHs were derived for different values of dynamic velocity of flow (Fig. 7). The obtained GUH with dynamic flow velocity of 0.68 m/s (approx.) shows closer agreement with SUH derived from CWC approach. Since, CWC approach is independent of climatic parameter (i.e. dynamic flow velocity) and geomorphologic characteristics other than the slope, drainage area and length of main stream; therefore, the resulting UH may have higher computational error. Hence, the GUH based approach will be superior to the CWC approach. Based on the analysis of the results, three equations (i.e eqs 24 through 27) were also developed for the estimation of peak ordinates of direct surface runoff hydrographs for Gomti basin provided the excess rainfall depth is available.

Concluding, it may be remarked that the proposed technique, which was not yet developed for the Gomti river basin of India will, helps the engineers for accurate estimation of the flood hydrograph as well as for the modeling of pollutants transport. Besides this, the described technique is cost-effective and quite accurate for determining the GUH and flood hydrograph for any catchment / basin (gauged or ungauged) as it requires DEM of the catchment that can be freely obtained from SRTM source.

Appendix-I: The Newton-Rapson method to estimate n

From eq. (23), the overall function can be defined as follows:

$$f = \frac{(n-1)}{\Gamma(n)} \exp[-(n-1)] \cdot (n-1)^{n-1} - 0.5764 \left[\frac{R_b}{R_a}\right]^{0.55} \times R_l^{0.05}$$
(28)

The first derivative of f with respect to n can be expressed as follows:

$$f' = \frac{1}{\Gamma(n)} \exp(-n+1) \cdot (n-1)^{n-1} - \frac{n-1}{\Gamma(n)} \exp(-n+1) \cdot (n-1)^{n-1} \cdot \psi(n) - \frac{n-1}{\Gamma(n)} \exp(-n+1) \cdot (n-1)^{n-1} + \frac{n-1}{\Gamma(n)} \exp(-n+1) \cdot (n-1)^{n-1} \times \{\ln(n-1)+1\}$$
(29)

where $\psi(n)$ is a **P**si function also known as digamma function and is expressed as follows:

$$\psi(z) = \frac{d}{dz} [\ln\{\Gamma(z)\}] = \frac{\Gamma'(z)}{\Gamma(z)}$$
(30)

The Psi function for any real number was evaluated using the following set of equation (eqs. 31a through 31d).

$$\psi(1) = -\gamma = -0.57722$$
 (31a)
 $\psi(1/2) = -\gamma - 2\ln(2) = -1.96351$ (31b)

$$\psi(n) = -\gamma + \sum_{k=1}^{n-1} k^{-1} \quad (n \ge 2)$$
 (31c)

$$\psi\left(n+\frac{1}{2}\right) = -\gamma - 2\ln(2) + 2\sum_{k=1}^{n} \left(\frac{1}{2k-1}\right) \quad (n \ge 1)$$
(31d)

The Gamma function of a dummy variable *z*, $\Gamma(z)$ can be defined as follows:

$$\Gamma(z) = \int_{0}^{\infty} t^{z-1} \cdot \exp(-t) dt$$
(32)

The optimization algorithm for Newton-Rapson method is expressed as follows:

$$n_{\rm new} = n_{\rm old} - \frac{f(n_{\rm old})}{f'(n_{\rm old})}$$
(33)

The optimization terminated when the following condition is achieved.

 $(n_n - new - n_old) \le 0.00001$ (34)

Appendix-II: CWC procedure of UH computation for Gomti river basin

Gomti basin falls under sub zone 1(f) of the hydro-meteorological homogeneity. Considering the sub zone 1(f), the relations established between physiographic and unit hydrograph (UH) parameters developed by Central Water Commission, India (CWC, 1985) are applied for derivation of 2- hour SUH for an ungauged catchment. The following steps are involved for derivation of 2 hour unit hydrograph. The

parameters used for developing SUH as proposed by CWC are given in the Fig. 8. Using the suggested procedure a 2 hour SUH has been developed for the Gomti river.



Fig. 8 Components of SUH for CWC procedure

Step 1: Analysis of 'catchment physiography'

Physiographic parameters viz. the catchment area A, length of the longest stream L and equivalent stream slope S are determined from the catchment area plan for the estimation of L/\sqrt{S} , where the equivalent stream slope S can be calculated as follows.

$$S = \frac{\sum L_i (D_{i-1} - D_i)}{L^2}$$
(35)

where L_i is the length of the i^{th} segment in km, D_{i-1} , D_i are the depths of the river at the point of intersection of (i-1) and i^{th} contours respectively from the base line (datum) drawn at the level of the point of study in meters and L is the length of the longest stream. For Gomti river basin equivalent slope is found to be 5.2 m /km

Step 2: Determination of 'peak value for 2-hour SUH'

The following equations is used for the peak discharge per unit of catchment area q_p (m³/s/sq km) for the

2-hour SUH of Gomti river basin.

$$q_p = 2.030/(L/\sqrt{S})^{0.649}$$
(36)
The peak discharge Q_p (m³/s) for the 2 hour SUH is obtained as follows

$$Q_p = q_p \times A \tag{37}$$

Step 3: Determination of 'time from center of effective rainfall duration to the peak value for 2-hour SUH'

The time to peak t_p (hr) for 2-hour SUH is estimated using the following relationship.

 $t_p = 1.858/(q_p)^{1.308}$ (38)

Step 4: Estimation of the ' time to peak (T_m) for 1-hour SUH'

The time to peak T_m for 2-hour SUH (Figure 8) is computed using the following relationship. (39) $T_{m} = t_{p} + t_{r} / 2$

Step 5: Estimation of ' time base for 2-hour SUH'

Time base T_{R} (hour) is computed as follows.

$$T_B = 7.744 \left(t_p \right)^{0.779} \tag{40}$$

The time width parameters (in hr) W_{50} , W_{75} , WR_{50} and WR_{75} (Figure 8) can be calculated using the

Step 6: Estimation of 'time widths for different % values of peak discharge of 2-hour SUH'

following set of equations.

$$W_{50} = 2.217 / (q_p)^{0.990}$$
 (41)
 $W_{75} = 1.447 / (q_p)^{0.876}$ (42)
 $WR_{50} = 0.812 / (q_p)^{0.907}$ (43)
 $WR_{75} = 0.606 / (q_p)^{0.791}$ (44)

(Note: the values in the parenthesis refer to Figure 8)

Step 7: Computation of 'unit depth'

Parameters viz. Q_n , T_m , t_n , T_B , W_{50} , WR_{50} , W_{75} , WR_{75} are used and a 2-hour SUH is generated. The runoff depth (d) under the SUH is computed as follows.

$$d = \frac{0.36 \times Q_i \times t_r}{A} \,\mathrm{cm} \tag{45}$$

In case, the depth of runoff (d) for the SUH is not equal to 1.0 cm, suitable modifications may be made by smoothening the graph so as to contain a unit volume (i.e., 1 cm \times catchment area).

Step 8: Cross checking of Time Base of SUH by 'triangular unit hydrograph (TUH) method'

Conventionally the time base T_B of a TUH is adopted as follows.

 $T_B = 2.67 \times T_m$

This T_{B} value comes out to be same as CWC method and therefore, the SUH derived from CWC method

is accepted for the Gomti river basin. Unit depth calculation for the derived SUH is done. Step 9: Conversion of 2 hr UH to 1 hr UH by S-Hydrograph method

To derive 1-hour SUH from 2-hour SUH, S-hydrograph technique was used.

(46)

D)

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R_b = bifurcation ratio N_i = number of *i*-th order streams Ω = highest stream order of the basin R_{i} = stream length ratio \overline{L}_i = average length of *i*-th order stream L_i = length of *i*-th order stream R_a = stream area ratio \overline{A}_{i} = average area of *i*-th order stream $A_{i,j}$ = total area that drains into the j^{th} stream of order *i* D_d = drainage density q_p = peak flow rate L_{Ω} = length of highest order stream I(n,t/k)= incomplete gamma function of order n at (t/k)Г = Gamma function $\psi(n)$ = Psi function of n $\Gamma'(z)$ = derivative of gamma function Α = drainage area of the basin С = constant for drainage basin similarity CWC = Central Water Commission D = duration of UH DEM = digital elevation model DRH = direct runoff hydrograph = exponential function exp GIS = geographic information system GIUH = geomorphologic instantaneous unit hydrograph GUH = geomorphologic unit hydrograph = fall in the basin h IUH = instantaneous unit hydrograph k = storage coefficient (Nash parameter) = horizontal distance over which fall (i.e. h) take place L Lb = length of the basin n = number of linear reservoir (Nash parameter) P_r = watershed perimeter = peak flow rate Q_p $= R_l / R_h$ r S = average slope of the basin SRTM =Shuttle Radar Topographic Mission SUH = synthetic unit hydrograph t = sampling time interval = time to peak

Notations & Abbreviations

t_p

TUH	= triangular unit hydrograph
U(D, t)	= ordinates of <i>D</i> hour UH
<i>u</i> (<i>t</i>)	= ordinate of IUH
UH	= unit hydrograph
V	= dynamic velocity of flow