

In cooperation with the Montana Department of Transportation

Evaluation of Pier-Scour Equations for Coarse-Bed Streams



Scientific Investigations Report 2004-5111

U.S. Department of the Interior
U.S. Geological Survey

COVER PHOTOGRAPH: Clark Fork at Petty Creek Road bridge near Alberton, Montana. Photograph by Katherine J. Chase, U.S. Geological Survey, taken on May 31, 2002.

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By Katherine J. Chase and Stephen R. Holnbeck

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U.S. Geological Survey**

U.S. Department of the Interior
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Conversion Factors, Acronyms, and Symbols

| Multiply | By | To obtain |
|---|---------|---------------------------|
| cubic foot per second (ft ³ /s) | 0.28317 | cubic meter per second |
| foot (ft) | 0.3048 | meter |
| foot per foot (ft/ft) | 1.0 | meter per meter |
| foot per second (ft/s) | 0.3048 | meter per second |
| inch (in.) | 25.9 | millimeter (mm) |
| pounds per cubic foot (lb/ft ³) | 16.02 | kilograms per cubic meter |
| square mile (mi ²) | 2.59 | square kilometer |

Acronyms used in this report:

| | |
|-------|---|
| BSDMS | Bridge Scour Data Management System, U.S. Geological Survey |
| CSU | Colorado State University |
| FHWA | Federal Highway Administration, U.S. Department of Transportation |
| MDT | Montana Department of Transportation |
| USGS | U.S. Geological Survey |

Symbols used in this report:

| | |
|-----------|---|
| b | Width of bridge pier, in ft |
| b_e | Width of bridge pier projected normal to the approach flow, in ft |
| c | Exponent in Simplified Chinese Equation for live-bed scour |
| D_x | Particle size for which “ x ” percent of bed material is finer, in mm or ft |
| D_m | Mean particle size of bed material, in mm or ft |
| Fr_1 | Froude Number directly upstream from the pier, dimensionless |
| K_s | Simplified pier-shape coefficient for Simplified Chinese equation, dimensionless |
| K_w | Correction factor for wide piers |
| K_1 | Correction factor for pier-nose shape, dimensionless |
| K_2 | Correction factor for angle of attack of flow at the pier, dimensionless |
| K_3 | Correction factor for bed condition, dimensionless |
| K_4 | Correction factor for coarse bed material, dimensionless |
| L | Pier length, in ft |
| V_c | Critical (incipient motion) velocity for the D_m -sized particle, in ft/s |
| V_{cx} | Critical (incipient motion) velocity for particles of size D_x , in ft/s |
| V_{ic} | Approach velocity that corresponds to critical velocity at the pier, in ft/s |
| V_{icx} | Approach velocity required to initiate scour at the pier for the grain size D_x , in ft/s |
| V_o | Approach velocity directly upstream from the pier, in ft/s |
| V_R | Velocity ratio, dimensionless |
| y_o | Depth of flow directly upstream from the pier, in ft |
| y_s | Depth of pier scour below ambient bed, in ft |
| θ | Angle of attack of the flow |
| ϕ | Coefficient based on the shape of the pier nose, dimensionless |

Evaluation of Pier-Scour Equations for Coarse-Bed Streams

by Katherine J. Chase and Stephen R. Holnbeck

Abstract

Streambed scour at bridge piers is among the leading causes of bridge failure in the United States. Several pier-scour equations have been developed to calculate potential scour depths at existing and proposed bridges. Because many pier-scour equations are based on data from laboratory flumes and from cohesionless silt- and sand-bottomed streams, they tend to overestimate scour for piers in coarse-bed materials. Several equations have been developed to incorporate the mitigating effects of large particle sizes on pier scour, but further investigations are needed to evaluate how accurately pier-scour depths calculated by these equations match measured field data.

This report, prepared in cooperation with the Montana Department of Transportation, describes the evaluation of five pier-scour equations for coarse-bed streams. Pier-scour and associated bridge-geometry, bed-material, and streamflow-measurement data at bridges over coarse-bed streams in Montana, Alaska, Maryland, Ohio, and Virginia were selected from the Bridge Scour Data Management System. Pier scour calculated using the Simplified Chinese equation, the Froehlich equation, the Froehlich design equation, the HEC-18/Jones equation and the HEC-18/Mueller equation for flood events with approximate recurrence intervals of less than 2 to 100 years were compared to 42 pier-scour measurements. Comparison of results showed that pier-scour depths calculated with the HEC-18/Mueller equation were seldom smaller than measured pier-scour depths. In addition, pier-scour depths calculated using the HEC-18/Mueller equation were closer to measured scour than for the other equations that did not underestimate pier scour. However, more data are needed from coarse-bed streams and from less frequent flood events to further evaluate pier-scour equations.

Introduction

Streambed scour at bridge piers is among the leading causes of bridge failure in the United States (Landers and Mueller, 1996, p.1). As a result, the Federal Highway Administration (FHWA) has developed methods for State highway agen-

cies to calculate potential scour depths at existing and proposed bridges (Richardson and Davis, 2001). Methods for calculating pier scour are based on empirical equations relating maximum scour depth to various hydraulic and bridge-geometry variables. Many of the empirical equations currently available (2004) are based on data from laboratory flumes and from cohesionless silt- and sand-bottom streams. Equation improvements continue to be made with the ultimate goal of minimizing underestimation and overestimation of scour. If pier scour is underestimated, scour depths assumed in the bridge design could be exceeded during large floods. Excess scour could lessen support for the bridge pier and destabilize the bridge. To ensure that pier-scour depth is not underestimated, some empirical equations have been adjusted to yield more conservative (larger) scour estimates. However, calculated scour depths from those equations might indicate that more bridges are scour-critical (subject to failure due to scour) than is actually the case and thus may lead to expensive over-design or unnecessary retrofitting of pier foundations. The goal for design of new bridges and analysis of existing structures is to ensure that bridge foundations withstand the effects of scour, but are not larger, deeper, or more expensive than necessary. Therefore, estimated scour depths from a pier-scour equation for a given set of site and flood conditions need to be as accurate as possible. However, when estimates are in error, scour needs to be overestimated rather than underestimated for safety considerations in bridge design.

To improve the understanding of scour processes and to develop more reliable pier-scour equations, the FHWA and many State highway agencies have cooperated with the U.S. Geological Survey (USGS) to collect onsite scour data at bridges. Data from these studies were analyzed by Landers and Mueller (1996) and used to evaluate 14 empirical pier-scour equations. Three hundred eighty-four pier-scour measurements at 56 bridges in 13 states were then compared to pier scour calculated by each of the 14 equations. Comparisons showed that none of the 14 equations accurately calculated scour for the full range of conditions measured in the field. Moreover, Landers and Mueller (1996, p. 111-112) found that the pier-scour equation recommended in the FHWA Hydrologic Engineering Circular 18, Second Edition (Richardson and others, 1993) overestimated pier scour for many measurements. This equation,

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referred to as the HEC-18 (2nd edition) equation in this report, did not account for the effects that coarse streambed material has on scour.

For coarse-bed streams, Landers and Mueller (1996, p. 95-119) identified several empirical equations that accounted for bed-material size, including the Simplified Chinese equation, the Froehlich equation, and the Froehlich design equation. Landers and Mueller (1996) cite Gao and others (1993) as the source for the Simplified Chinese equation and Froehlich (1988) as the source for the Froehlich and Froehlich design equations. In the Landers and Mueller study (1996, p. 109-111), the Simplified Chinese equation and the Froehlich equation frequently underestimated pier scour, while the Froehlich design equation only rarely underestimated pier scour.

Since publication of the HEC-18 (2nd edition) (Richardson and others, 1993), a third edition (Richardson and Davis, 1995) and a fourth edition (Richardson and Davis, 2001) have been published. The HEC-18 equation in the third edition included a pier-scour correction factor (K_4) for coarse bed material developed by J. Sterling Jones, Federal Highway Administration. The fourth edition included a modified K_4 factor that better accounted for streambed armoring in coarse-bed channels. Application of this modified K_4 factor was expected to result in substantially smaller and more reliable estimates of pier-scour depths in coarse-bed streams (Mueller, 1996).

The third- and fourth-edition versions of the HEC-18 equation are hereinafter referred to in this report as the HEC-18/Jones equation and the HEC-18/Mueller equation, respectively. Because of a small difference in the equation for critical velocity, the HEC-18/Mueller equation used in this report (Mueller, 1996, p. 158-160) is slightly different from the HEC-18/Mueller equation included in Richardson and Davis (2001, p. 6.6).

Because they account for bed-material size, and in some instances streambed armoring, the Simplified Chinese, Froehlich, Froehlich design, HEC-18/Jones, and HEC-18/Mueller equations are generally considered appropriate for calculating pier scour in coarse-bed streams in mountain states like Montana. However, the five equations have not been evaluated for use in coarse-bed streams. Therefore, the USGS and the Montana Department of Transportation (MDT) initiated a cooperative study in July 2000 to evaluate the five pier-scour equations by comparing calculated pier scour with measured pier scour from coarse-bed streams.

Purpose and Scope

This report describes the results of an evaluation of five pier-scour equations for coarse-bed streams. Scour depths calculated by the equations were compared with 18 pier-scour measurements at 3 bridge sites in Montana, 10 measurements at 2 sites in Alaska, 4 measurements at 1 site in Maryland, 1 measurement in Ohio, and 9 measurements at 2 sites in Virginia (fig. 1 and table 1). Scour measurements for each site are summarized in tables 1 and 2. The Simplified Chinese equation, the Froehlich equation, the Froehlich design equation, the HEC-18/Jones equation, and the HEC-18/Mueller equation

were evaluated based on the accuracy of pier-scour estimates and number and magnitude of underestimates resulting from each equation.

The definition of coarse-bed streams has evolved. Richardson and Davis (1995, p. 38) indicated that the pier-scour correction factor for coarse-bed streams (K_4) should be used only where D_{50} (the particle size for which 50 percent is finer) is greater than 60 mm. Later, Richardson and Davis (2001) characterized coarse-bed streams as those where D_{50} is greater than 40 mm. They further indicated that application of the K_4 factor was appropriate where D_{50} is greater than 2 mm and D_{95} is greater than 20 mm. In this report, coarse-bed streams were considered to be those having D_{50} greater than 50 mm. Thus, all sites analyzed easily met the more recent criterion for coarse-bed streams given by Richardson and Davis (2001).

Description of Pier-Scour Data Used to Evaluate Equations

Coarse-bed ($D_{50} > 50$ mm) pier-scour data compiled for this study were selected from the USGS Bridge Scour Data Management System (BSDMS) (Chad R. Wagner, U.S. Geological Survey, written commun., 2002). The selected BSDMS data included 15 pier-scour measurements from two Montana sites: Gallatin River at U.S. 191 near Gallatin Gateway (site 32), and Yellowstone River at U.S. 89, near Emigrant (site 33) (tables 1 and 2). The data selected from the BSDMS also included measurements (not previously analyzed by other researchers) made at site 33 during 1996 and 1997, when both peak discharges were close to the 50-year and 100-year floods, respectively. Furthermore, scour data for the Bitterroot River at U.S. 93 near Darby, Mont. (site 85), were recently added to the BSDMS and are included in this study.

This study also used selected BSDMS data from four other States. Included were 10 measurements from 2 rivers in Alaska—the Susitna River (site 1) and the Tazlina River (site 4); 4 measurements from the Youghiogheny River in Maryland (site 23); 1 measurement from the Little Miami River in Ohio (site 44); and 9 measurements from 2 rivers in Virginia—the Tye River (site 53) and Reed Creek (site 55).

Methods of Data Collection

Scour measurements in Montana were made using standard USGS sounding equipment that included either a four-wheel base or a bridgeboard device with a cable-suspended Columbus sounding weight attached to an A- or B-type sounding reel deployed from the upstream side of each bridge. Initial soundings typically were made for the entire cross section at the upstream face of the bridge before the runoff period to obtain baseline data. Velocity measurements were made using a vertical-axis current meter (Rantz, 1982) at several stations on both sides of each pier, outside the zone where flow typically

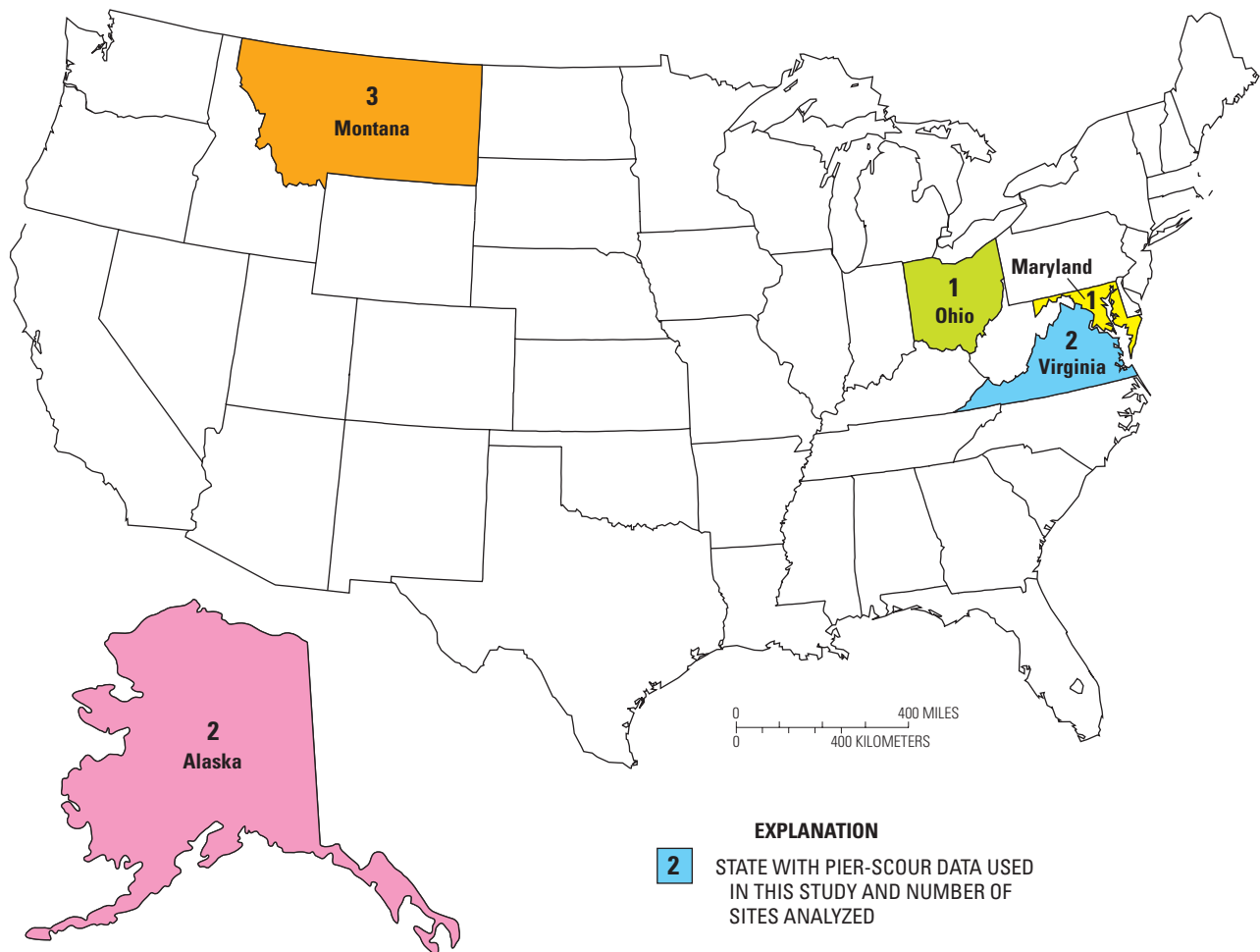


Figure 1. States from which pier-scour data were compiled.

accelerates near the pier. Sounding depths were related to a surveyed water-surface elevation, which was then related to a nearby vertical datum. Scour and velocity measurements at sites in the other States were made in a similar manner. Once the baseline cross-section data were plotted, the reference surface (Landers and Mueller, 1993, p. 2075-2080) was estimated and used with the scour soundings to derive the estimated scour depth. When more than one high-discharge scour measurement was made at a bridge, each was considered to be an independent measurement.

Bed material for each Montana site was characterized by particle counts (Wolman, 1954) performed near the bridge piers. Though the precision involved in determining bed-material size depends on a number of factors related to size distribution, spatial variation, sample size, and technique (Wolman, 1954; Kellerhals and Bray, 1971; Hey and Thorne, 1983), values are typically reported to two or three significant figures. Methods used to estimate bed-material sizes at sites outside of

Montana varied, but are believed to be consistent with those applied in Montana.

Comparison of Data Sets Used to Develop and Evaluate Equations

Though researchers in several States are working to collect, compile, and analyze bridge-scour data, the database containing pier-scour measurements and associated site and flow information for coarse-bed streams is fairly small. Therefore, an evaluation database that is totally independent of the database used to develop the pier-scour equations was unavailable for this study. In this study, 17 of the 42 measurements used to evaluate pier-scour equations also were used to develop the HEC-18/Mueller equation (D.S. Mueller, U.S. Geological Survey, written commun., 2002). However, 25 of the 42 pier-scour measurements used in this evaluation were not used in the

Table 1. Hydrologic and hydraulic data for selected pier-scour sites

[Drainage area and channel-slope data for site 85 from Bridge Scour Database Management System. Area and slope data for other bridges from Landers and Mueller (1996). Discharge measurement data from Bridge Scour Database Management System, unless otherwise noted. Abbreviations: BSDMS, Bridge Scour Database Management System; mi², square miles; ft/ft, foot per foot; ft³/s, cubic foot per second; NA, not applicable; Q_x , approximate peak discharge at the x -year recurrence interval, in ft³/s; S.R., State Road; U.S., United States; USGS, U.S. Geological Survey. Symbol: <, less than; --, data not available]

| BSDMS site number | State | Bridge location | Site identification number of nearest USGS gaging station | Drainage area (mi ²) | Channel slope (ft/ft) | Discharge, in ft ³ /s, for recurrence interval, in years | | | | | Date of scour measurement | Discharge during scour measurement (ft ³ /s) | Approximate recurrence interval (years) |
|-------------------|----------|--|---|----------------------------------|-----------------------|---|----------|----------|-----------|-----------|---------------------------|---|---|
| | | | | | | Q_2 | Q_{10} | Q_{50} | Q_{100} | Q_{500} | | | |
| 32 ¹ | Montana | Gallatin River at U.S. 191, near Gallatin Gateway | 06043500 | 825 | 0.0063 | 5,340 | 7,930 | 9,880 | 10,700 | 12,300 | 06/06/91 | 6,420 | 5 |
| | | | | | | | | | | | 06/18/92 | 2,930 | <2 |
| | | | | | | | | | | | 06/23/93 | 3,360 | <2 |
| 33 ² | Montana | Yellowstone River at U.S. 89, near Emigrant | NA | 2,844 | .0022 | 18,100 | 25,800 | 31,400 | 33,400 | 38,000 | 05/21/93 | 17,600 | <2 |
| | | | | | | | | | | | 05/27/93 | 17,100 | <2 |
| | | | | | | | | | | | 06/30/93 | 8,570 | <2 |
| | | | | | | | | | | | 06/12/96 | 31,900 | 50 |
| | | | | | | | | | | | 06/09/97 | 33,300 | 100 |
| 85 ¹ | Montana | Bitterroot River near Darby | 12344000 | 1,049 | .0038 | 5,890 | 9,790 | 12,700 | 13,900 | 16,300 | 06/11/96 | 8,787 | 5 |
| 1 ³ | Alaska | Susitna River near Sunshine | 15292780 | 11,500 | .0004 | -- | -- | -- | 186,000 | 206,000 | 07/02/71 | 74,600 | -- |
| | | | | | | | | | | | 08/11/71 | 171,000 | ⁴ 20 |
| 4 ³ | Alaska | Tazlina River at Richardson Highway, near Glennallen | 15202000 | 2,670 | .0021 | -- | -- | -- | 79,400 | 109,000 | 09/02/71 | 25,000 | ⁴ 6 |
| | | | | | | | | | | | 09/04/71 | 39,400 | ⁴ 6 |
| 23 ⁵ | Maryland | Youghiogheny River at S.R. 42, at Friendsville | 03076500 | 295 | .0050 | 6,250 | 10,300 | 13,200 | 14,400 | -- | 07/13/90 | ⁶ 6,680 | -- |
| | | | | | | | | | | | 04/01/93 | ⁶ 4,410 | -- |
| 44 | Ohio | Little Miami River at S.R. 350, at Fort Ancient | 03242500 | 675 | .0008 | -- | -- | -- | -- | -- | 12/19/90 | 4,620 | ⁴ 2 |
| 53 ⁷ | Virginia | Tye River at S.R. 56, near Lovingson | 02027000 | 93 | .0029 | 3,540 | 9,170 | 17,800 | 23,000 | 39,800 | 05/03/89 | ⁸ 866 | -- |
| | | | | | | | | | | | 05/07/89 | ⁸ 1,250 | -- |
| | | | | | | | | | | | 04/22/92 | ⁸ 3,070 | -- |
| 55 | Virginia | Reed Creek at S.R. 649, near Wytheville | 03166700 | -- | .0001 | -- | -- | -- | -- | -- | -- | -- | -- |

¹Flood-frequency data based on annual peak discharges 1890-1998 (Parrett and Johnson, 2004).

²Flood-frequency data for gaging station 06191500, Yellowstone River at Corwin Springs, Mont., (Charles Parrett, written commun., 2002) adjusted for drainage area at bridge (1890-1998).

³Flood-frequency data from Heinrichs and others (2001).

⁴Approximate recurrence interval reported in the Bridge Scour Data Management System.

⁵Flood-frequency data for water years 1940-1979 (Carpenter, 1983, p. 187).

⁶Instantaneous peak discharge (Charles J. Strain, U.S. Geological Survey, written commun., 2002); could be different from discharge at time of scour measurement.

⁷Flood-frequency data based on weighted average of log-Pearson type III analysis and regional regression (Bisesse, 1995, p. 49).

⁸Instantaneous peak discharge (Hayes, 1996, p. 28); could be different from discharge at time of scour measurement.

Table 2. Summary of pier-scour measurements

[Pier identification: number or location of pier at which measurement took place. Scour condition: refers to general condition of the bed-material movement upstream from the bridge during the time of measurement—clear, bed material not in motion; live, bed material generally in motion; *, scour condition based on comparison of measured velocity and computed incipient-motion velocity, rather than observation or hydrologist’s judgement. Pier shape: sharp-nosed, round-nosed. Pier skew: measured in degrees from line parallel to flow. Measurement error: scour measurement error (in feet) estimated by hydrologist. For example, for measurement number 228 the hydrologist measured 0.8 ft of scour and estimated a measurement error of 0.3 ft. Therefore, the measured scour depth could vary from 0.5 ft to 1.1 ft. Abbreviations: no., number; *L*, length; ft, foot; deg, degrees; *b*, width; *V*, velocity; ft/s, foot per second; mm, millimeters. Symbol: --, not available]

| BSDMS ¹ site no. | Measure- ment no. ² | Date | Pier identi- fication | Scour con- dition | Pier shape | Pier <i>L</i> (ft) | Pier skew (deg) | Pier <i>b</i> (ft) | <i>V</i> (ft/s) | Flow depth (ft) | Particle size for which indicated percentage of bed material is finer ³ | | | Scour depth (ft) | Meas- ure- ment error (ft) |
|-----------------------------------|--------------------------------------|----------|-----------------------------|-------------------------|---------------|--------------------------|--------------------|--------------------------|--------------------|--------------------|--|--------------------------------|--------------------------------|------------------------|--|
| | | | | | | | | | | | <i>D</i> ₅₀ (mm) | <i>D</i> ₉₀ (mm) | <i>D</i> ₉₅ (mm) | | |
| Montana | | | | | | | | | | | | | | | |
| 32 | 228 | 06/06/91 | P1 | Clear* | Sharp | 39.3 | 3 | 3.4 | 8.4 | 4.8 | 95 | 300 | 330 | 0.8 | 0.3 |
| 32 | 229 | 06/18/92 | P1 | Clear* | Sharp | 39.3 | 3 | 3.4 | 5.1 | 3.3 | 95 | 300 | 330 | 1.2 | .3 |
| 32 | 230 | 06/23/93 | P1 | Clear* | Sharp | 39.3 | 3 | 3.4 | 6.2 | 3.4 | 95 | 300 | 330 | 1.9 | .3 |
| 32 | 231 | 06/06/91 | P2 | Clear* | Sharp | 39.3 | 3 | 3.4 | 10.6 | 5.5 | 95 | 300 | 330 | 5.5 | .5 |
| 32 | 232 | 06/18/92 | P2 | Clear* | Sharp | 39.3 | 3 | 3.4 | 7.0 | 3.7 | 95 | 300 | 330 | 4.6 | .5 |
| 32 | 233 | 06/23/93 | P2 | Clear* | Sharp | 39.3 | 3 | 3.4 | 7.0 | 3.8 | 95 | 300 | 330 | 4.5 | .5 |
| 33 | 234 | 05/21/93 | P1 | Clear* | Sharp | 34.0 | 0 | 3.1 | 8.0 | 8.7 | 73 | 180 | 190 | 2.5 | .5 |
| 33 | 235 | 05/27/93 | P1 | Clear* | Sharp | 34.0 | 0 | 3.1 | 8.2 | 8.3 | 73 | 180 | 190 | 2.3 | .5 |
| 33 | 236 | 06/30/93 | P1 | Clear* | Sharp | 34.0 | 0 | 3.1 | 4.9 | 6.6 | 73 | 180 | 190 | 1.9 | .5 |
| 33 | 237 | 05/21/93 | P2 | Clear* | Sharp | 34.0 | 0 | 3.2 | 7.6 | 8.2 | 73 | 180 | 190 | 1.6 | .3 |
| 33 | 238 | 05/27/93 | P2 | Clear* | Sharp | 34.0 | 0 | 3.1 | 8.0 | 7.8 | 73 | 180 | 190 | 1.8 | .3 |
| 33 | 239 | 06/30/93 | P2 | Clear* | Sharp | 34.0 | 0 | 3.2 | 4.8 | 6.2 | 73 | 180 | 190 | 1.1 | .3 |
| 33 | 240 | 05/21/93 | P3 | Clear* | Sharp | 34.0 | 0 | 3.1 | 3.3 | 7.4 | 73 | 180 | 190 | .3 | .3 |
| 33 | 241 | 05/27/93 | P3 | Clear* | Sharp | 34.0 | 0 | 3.1 | 3.6 | 6.8 | 73 | 180 | 190 | .4 | .3 |
| 33 | 242 | 06/30/93 | P3 | Clear* | Sharp | 34.0 | 0 | 3.1 | 3.5 | 6.0 | 73 | 180 | 190 | .4 | .3 |
| 33 | -- | 06/12/96 | P1 | Clear* | Sharp | 34.0 | 0 | 3.1 | 9.7 | 9.8 | 73 | 180 | 190 | 4.0 | .5 |
| 33 | -- | 06/09/97 | P1 | Clear* | Sharp | 34.0 | 0 | 3.1 | 9.1 | 9.8 | 73 | 180 | 190 | 4.0 | .5 |
| 85 | -- | 06/11/96 | P2 | Clear* | Sharp | -- | 0 | 3.0 | 8.5 | 6.6 | 56 | 88 | 110 | 2.3 | .5 |
| Alaska | | | | | | | | | | | | | | | |
| 1 | 1 | 07/02/71 | P1 | Clear | Sharp | 20.0 | 0 | 5.0 | 6.5 | 19.0 | 70 | 90 | 96 | 2.5 | .5 |
| 1 | 2 | 08/11/71 | P1 | Clear | Sharp | 20.0 | 0 | 5.0 | 10.0 | 17.5 | 70 | 90 | 96 | 2.0 | .5 |
| 1 | 3 | 07/02/71 | P2 | Clear | Sharp | 20.0 | 0 | 5.0 | 8.5 | 13.5 | 70 | 90 | 96 | 2.5 | .5 |
| 1 | 4 | 08/11/71 | P2 | Clear | Sharp | 20.0 | 0 | 5.0 | 9.5 | 21.5 | 70 | 90 | 96 | 2.0 | 1.0 |
| 1 | 5 | 07/02/71 | P3 | Clear | Sharp | 20.0 | 0 | 5.0 | 7.0 | 11.0 | 70 | 90 | 96 | 2.0 | .5 |
| 1 | 6 | 08/11/71 | P3 | Clear | Sharp | 20.0 | 0 | 5.0 | 11.5 | 17.0 | 70 | 90 | 96 | 2.0 | 1.0 |
| 1 | 7 | 07/02/71 | P4 | Clear | Sharp | 20.0 | 0 | ⁴ 5.0 | 5.0 | 13.5 | 70 | 90 | 96 | 5.0 | .5 |

Table 2. Summary of pier-scour measurements—Continued

| BSDMS ¹ site no. | Measure- ment no. ² | Date | Pier identi- fication | Scour con- dition | Pier shape | Pier <i>L</i> (ft) | Pier skew (deg) | Pier <i>b</i> (ft) | <i>V</i> (ft/s) | Flow depth (ft) | Particle size for which indicated percentage of bed material is finer ³ | | | Scour depth (ft) | Meas- ure- ment error (ft) |
|-----------------------------------|--------------------------------------|----------|-----------------------------|-------------------------|---------------|--------------------------|--------------------|--------------------------|--------------------|--------------------|--|--------------------------------|--------------------------------|------------------------|--|
| | | | | | | | | | | | <i>D</i> ₅₀ (mm) | <i>D</i> ₉₀ (mm) | <i>D</i> ₉₅ (mm) | | |
| Alaska--continued | | | | | | | | | | | | | | | |
| 1 | 8 | 08/11/71 | P4 | Clear | Sharp | 20.0 | 0 | ⁴ 5.0 | 9.5 | 17.5 | 70 | 90 | 96 | 5.0 | 1.0 |
| 4 | 29 | 09/02/71 | P1 | Live | Round | -- | 0 | ⁵ 15 | 9.5 | 12.0 | 90 | 130 | 140 | 5.0 | .5 |
| 4 | 30 | 09/04/71 | P1 | Live | Round | -- | 0 | ⁵ 15 | 11.5 | 15.0 | 90 | 130 | 140 | 5.5 | .5 |
| Maryland | | | | | | | | | | | | | | | |
| 23 | 153 | 07/13/90 | Left | Clear* | Sharp | 41.7 | 0 | 5.0 | 7.7 | 7.9 | 110 | 290 | 350 | 1.1 | 1.0 |
| 23 | 154 | 04/01/93 | Left | Clear* | Sharp | 41.7 | 0 | 5.0 | 6.8 | 6.8 | 110 | 290 | 350 | 1.4 | 1.0 |
| 23 | 155 | 07/13/90 | Right | Clear* | Sharp | 41.7 | 0 | 5.0 | 8.6 | 9.9 | 110 | 290 | 350 | 2.7 | 1.0 |
| 23 | 156 | 04/01/93 | Right | Clear* | Sharp | 41.7 | 0 | 5.0 | 6.2 | 8.0 | 110 | 290 | 350 | 1.7 | 1.0 |
| Ohio | | | | | | | | | | | | | | | |
| 44 | 288 | 12/19/90 | P2 | Clear* | Round | 24.3 | 0 | 2.5 | 3.7 | 5.6 | 60 | 74 | 75 | .7 | .3 |
| Virginia | | | | | | | | | | | | | | | |
| 53 | 358 | 05/03/89 | P2 | Clear* | Round | 41.0 | 0 | 2.0 | 1.8 | 1.5 | 72 | 220 | 250 | .8 | 1.0 |
| 53 | 359 | 05/07/89 | P2 | Live* | Round | 41.0 | 0 | 2.0 | 5.1 | 2.2 | 72 | 220 | 250 | .6 | 1.0 |
| 53 | 360 | 04/22/92 | P2 | Live* | Round | 41.0 | 0 | 2.0 | 5.2 | 5.5 | 72 | 220 | 250 | 1.6 | 1.0 |
| 53 | 361 | 05/03/89 | P3 | Live* | Round | 41.0 | 0 | 2.0 | 4.0 | 4.0 | 72 | 220 | 250 | 1.0 | 1.0 |
| 53 | 362 | 05/07/89 | P3 | Live* | Round | 41.0 | 0 | 2.0 | 5.3 | 5.0 | 72 | 220 | 250 | 1.2 | 1.0 |
| 53 | 363 | 04/22/92 | P3 | Live* | Round | 41.0 | 0 | 2.0 | 8.5 | 8.6 | 72 | 220 | 250 | 2.5 | 1.0 |
| 55 | 376 | 03/29/91 | P2 | Clear | Round | 30.0 | 0 | 2.0 | 3.7 | 2.5 | 55 | 95 | 110 | 1.5 | 1.0 |
| 55 | 377 | 06/05/92 | P2 | Clear | Round | 30.0 | 0 | 2.0 | 5.5 | 10.5 | 55 | 95 | 110 | 2.1 | 1.0 |
| 55 | 378 | 03/24/93 | P2 | Clear | Round | 30.0 | 0 | 2.0 | 6.4 | 10.5 | 55 | 95 | 110 | 1.8 | 1.0 |

¹Data compiled from U.S. Geological Survey, Bridge Scour Data Management System (BSDMS) or from Landers and Mueller (1996, p. 58-66).

²Measurement number in Landers and Mueller (1996).

³Where *D*₉₀ or *D*₉₅ were not available, they were calculated from $D_{90} = D_{50} (D_{84}/D_{50})^{1.282}$ and $D_{95} = D_{50} (D_{84}/D_{50})^{1.645}$.

⁴Substantial debris on pier could lead to underestimation of scour depths.

⁵Maximum pier width in Landers and Mueller (1996).

HEC-18/Mueller equation development. Therefore, a substantial portion of the data used in this study could be considered independent for evaluating the HEC-18/Mueller equation. The overlap between data used in this evaluation and in development of each of the other four equations is unknown, but is probably smaller because the other equations are less recent. In addition, this study focused on the subset of pier-scour data pertaining to coarse-bed streams. Data used for evaluation of the equations included only data from streams where the bed-material D_{50} was greater than 50 mm, with an average D_{50} equal to 76 mm. In contrast, the data used to develop the HEC-18/Mueller equation included D_{50} values as small as 0.18 mm, with an average D_{50} equal to 12 mm.

Few pier-scour data are available for rare floods in coarse-bed streams. Data used in this study were collected during floods substantially smaller than the 100-year event, with the notable exception of the Yellowstone River (table 1). However, predicted scour depths for the 100-year and even 500-year floods typically are required for bridge design. More scour measurements from flows closer to the 100-year and 500-year events are needed in order to test the performance of the scour-prediction equations for these less common events.

Description of Pier-Scour Equations for Coarse-Bed Streams

The five pier-scour equations evaluated in this report include the Simplified Chinese equation, the Froehlich equation, the Froehlich design equation, the HEC-18/Jones equation, and the HEC-18/Mueller equation. All of these equations attempt to account for the effects of the bed-material size on scour. The equations have been modified from their original format so that all units are in the foot-pound-second or English unit convention.

Simplified Chinese Equation

The Simplified Chinese pier-scour equation is based on laboratory and field data from China (Landers and Mueller, 1996, p. 98-100). This equation has different forms depending upon whether the scour condition is live-bed scour (bed material upstream from bridge is in motion) or clear-water scour (bed material upstream from bridge is not in motion). The Simplified Chinese equation for clear-water pier scour is defined as:

$$y_s = 1.141K_s b^{0.6} y_o^{0.15} D_m^{-0.07} \left(\frac{V_o - V_{ic}}{V_c - V_{ic}} \right) \quad (1)$$

where:

- y_s is the depth of pier scour below ambient bed, in feet;
- K_s is the simplified pier-shape coefficient:
 - $K_s = 1.0$ for cylinders,
 - $= 0.8$ for round-nosed piers,
 - $= 0.66$ for sharp-nosed piers;

- b is the width of bridge pier, in feet;
- y_o is the depth of flow directly upstream from the pier, in feet;
- D_m is the mean particle size of the bed material, in feet (for this study D_{50} was used for D_m);
- V_o is the approach velocity directly upstream from the pier, in feet per second;
- V_c is the critical (incipient motion) velocity, in feet per second, for the D_m -sized particle. If the density of water is assumed to be 62.4 pounds per cubic foot, and the bed material is assumed to have a specific gravity of 2.65, the equation for V_c can be expressed as:

$$V_c = 3.28 \left(\frac{y_o}{D_m} \right)^{0.14} \left(8.85 D_m + 6.05 E^{-7} \left[\frac{10 + 0.3048 y_o}{(0.3048 D_m)^{0.72}} \right] \right)^{0.5} \quad (1a)$$

- V_{ic} is the approach velocity, in feet per second, corresponding to critical velocity at the pier. V_{ic} can be calculated using the following equation:

$$V_{ic} = 0.645 \left(\frac{D_m}{b} \right)^{0.053} V_c \quad (1b)$$

The Simplified Chinese Equation for live-bed scour (when $V_o > V_c$) is expressed as:

$$y_s = 0.950 K_s b^{0.6} y_o^{0.15} D_m^{-0.07} \left(\frac{V_o - V_{ic}}{V_c - V_{ic}} \right)^c \quad (2)$$

where the exponent c is calculated using the following equation:

$$c = \left(\frac{V_c}{V_o} \right)^{8.20 + 2.23 \log D_m} \quad (2a)$$

To determine whether the equation for clear-water scour (equation 1) or live-bed scour (equation 2) was appropriate, critical velocity (V_c) was calculated using equation 1a. Pier scour was calculated using equation 1 for measurements where $V_o < V_c$ and equation 2 for measurements where $V_o > V_c$, regardless of the scour condition reported in the BSDMS.

Froehlich Equation

The Froehlich equation (Froehlich, 1988) was derived using regression analyses of pier-scour data from several investigations (Landers and Mueller, 1996, p. 101) and is defined as:

$$y_s = 0.32 b \phi F r_1^{0.2} \left(\frac{b_e}{b} \right)^{0.62} \left(\frac{y_o}{b} \right)^{0.46} \left(\frac{b}{D_{50}} \right)^{0.08} \quad (3)$$

where:

- y_s , y_o , and b are as previously defined;

8 Evaluation of Pier-Scour Equations for Coarse-Bed Streams

ϕ is a dimensionless coefficient based on the shape of the pier nose, as follows:

$$\begin{aligned}\phi &= 1.3 \text{ for square nosed-piers,} \\ &= 1.0 \text{ for round-nosed piers,} \\ &= 0.7 \text{ for sharp-nosed piers;}\end{aligned}$$

Fr_1 is the Froude Number directly upstream from the pier, dimensionless;

b_e is the width of the bridge pier projected normal to the approach flow, in feet; and

D_{50} is the particle size for which 50 percent of the bed material is finer, in feet.

According to Landers and Mueller (1996, p. 101) the Froehlich equation is based on field measurements that were presumed to have been made under live-bed scour conditions. However, in this study, the equation also was applied at all of the sites, even though clear-water scour was reported for many measurements.

Froehlich Design Equation

A scour-depth estimation method based on regression analysis, where underestimates are as likely as overestimates, is undesirable for bridge design because underestimation of scour depth is not acceptable. Froehlich (1988) found that by adding pier width (b) to the scour depth computed by equation 3, pier scour was not underestimated for any of the bridges in the data set used. Thus, the Froehlich design equation (Mueller, 1996, p. 102) is defined as:

$$y_s = 0.32b\phi Fr_1^{0.2} \left(\frac{b_e}{b}\right)^{0.62} \left(\frac{y_o}{b}\right)^{0.46} \left(\frac{b}{D_{50}}\right)^{0.08} + b \quad (4)$$

where all variables are as previously defined. The Froehlich design equation is included as a pier-scour calculation option within the computer model HEC-RAS, Version 3.1 (Brunner, 2002).

HEC-18/Jones Equation

The HEC-18/Jones equation is based on the Colorado State University (CSU) equation (Richardson and others, 1993). When compared with data from field measurements (Richardson and others, 1993), the CSU equation was found to more reliably calculate pier scour when compared to several other equations. The HEC-18/Jones equation incorporates a correction factor K_4 to account for armoring of the scour hole. Richardson and Davis (1995, p. 36-38) define the HEC-18/Jones equation as:

$$y_s = 2.0y_o K_1 K_2 K_3 K_4 \left(\frac{b}{y_o}\right)^{0.65} Fr_1^{0.43} \quad (5)$$

where:

y_s and y_o are as previously defined;

K_1 is the correction factor for pier-nose shape, dimensionless;

K_2 is the correction factor for angle of attack of flow at the pier, dimensionless;

K_3 is the correction factor for bed condition, dimensionless;

K_4 is the correction factor for armoring of coarse-bed material, dimensionless; and

b and Fr_1 are as previously defined.

The K_1 , K_2 , and K_3 correction factors are defined in table 3. K_4 is calculated by the following equation:

$$K_4 = [1 - 0.89(1 - V_R)^2]^{0.5} \quad (6)$$

where:

V_R is the velocity ratio, dimensionless, calculated as:

$$V_R = \left[\frac{V_o - V_{i50}}{V_{c90} - V_{i50}} \right] \quad (6a)$$

V_o is as previously defined;

V_{i50} is the approach velocity, in feet per second, required to initiate scour at the pier for the particle size D_{50} . V_{i50} is calculated as follows:

$$V_{i50} = 0.645 \left(\frac{D_{50}}{b}\right)^{0.053} V_{c50} \quad (6b)$$

D_{50} is as previously defined,

V_{c50} is the critical velocity, in feet per second, for incipient motion of the particle size D_{50} , and is further defined as follows:

$$V_{c50} = 11.21y_o^{1/6} D_{50}^{1/3} \quad (6c)$$

V_{c90} is the critical velocity, in feet per second, for incipient motion of the particle size D_{90} , and is calculated as follows:

$$V_{c90} = 11.21y_o^{1/6} D_{90}^{1/3} \quad (6d)$$

D_{90} is the particle size for which 90 percent of the bed material is finer, in feet.

Mueller and Wagner (in press) report that at sites where only D_{50} and D_{84} are determined, D_{90} can be calculated by:

$$D_{90} = D_{50} \left(\frac{D_{84}}{D_{50}}\right)^{1.282} \quad (6e)$$

where the ratio $\frac{D_{84}}{D_{50}}$ is referred to as the gradation coefficient (Mueller, 1996).

Table 3. K_1 , K_2 , and K_3 correction factors for the HEC-18/Jones equation

[Information in table from Richardson and Davis (1995). Abbreviations: b , pier width, in feet; ft, feet; L , pier length, in feet. Symbols: $>$, greater than; \geq , greater than or equal to]

| Correction Factor | Value or Equation |
|--|---|
| K_1: Pier-nose shape | |
| Square nose | 1.1 |
| Round nose | 1.0 |
| Circular cylinder | 1.0 |
| Group of cylinders | 1.0 |
| Sharp nose | .9 |
| K_2: Angle of attack of flow (θ) | |
| | $[\text{Cos}\theta + (L/b) \text{Sin}\theta]^{0.65}$, where θ = angle of attack of flow If $L/b > 12$, use 12 as a maximum |
| K_3: Bed condition | |
| Clear-water scour | 1.1 |
| Live-bed scour: | |
| Plane-bed and antidune bedform | 1.1 |
| Small dunes, 3 ft > dune height \geq 0.6 ft | 1.1 |
| Medium dunes, 9 ft > dune height \geq 3 ft | 1.1 to 1.2 |
| Large dunes, dune height > 9 ft | 1.3 |

Equation 6 is based on research indicating that when the approach velocity (V_o) is too low to move the D_{90} size of the bed material, scour depth is reduced (Richardson and Davis, 1995, p. 37). In this situation ($V_o < V_{c90}$, equation 6a), K_4 will be less than 1.0. However, for comparison purposes in this study, K_4 was allowed to be less than 1.0 even when $V_o > V_{i90}$. Richardson and Davis (1995, p. 38) recommend a minimum value of 0.7 for the HEC18/Jones K_4 correction factor and indicate that when the velocity ratio (V_R) exceeds 1.0, K_4 should default to 1.0. Richardson and Davis (1995, p. 38) also suggest that the HEC-18/Jones equation be applied only to sites where D_{50} is greater than 60 mm. However, the equation also was used in this study for four scour measurements from sites 55 and 85 where the bed-material D_{50} was estimated to be between 50 mm and 60 mm.

HEC-18/Mueller Equation

Mueller (1996, p. 160) proposed a modified K_4 correction factor for the HEC-18 equation:

$$K_4 = 0.4 \left(\frac{V_o - V_{i50}}{V_{c50} - V_{i95}} \right)^{0.15} \quad (7)$$

where V_{i95} is the approach velocity, in feet per second, required to initiate scour at the pier for the D_{95} particle size, in feet, and all other terms are as previously defined. Mueller (1996, p. 160) suggests an equation for critical velocity for incipient motion of

the D_{50} sized particles (V_{c50}) that is slightly different from the V_{c50} used in the HEC-18/Jones equation. Mueller's equation for V_{c50} varies in format for different D_{50} sizes (Mueller, 1996, p. 160). For D_{50} greater than 20 mm, V_{c50} is calculated as follows:

$$V_{c50} = 12.21 y_o^{1/6} D_{50}^{1/3} \quad (7a)$$

where all terms are as previously defined.

The V_{i95} term is calculated like V_{i50} in equation 6b as follows:

$$V_{i95} = 0.645 \left(\frac{D_{95}}{b} \right)^{0.053} V_{c95} \quad (7b)$$

V_{c95} is calculated similarly to V_{c50} in equation 7a as follows:

$$V_{c95} = 12.21 y_o^{1/6} D_{95}^{1/3} \quad (7c)$$

where D_{95} is the particle size for which 95 percent of the bed material is finer, in feet.

Mueller and Wagner (in press) determined that when information about D_{95} is not available, D_{95} can be calculated by:

$$D_{95} = D_{50} \left(\frac{D_{84}}{D_{50}} \right)^{1.645} \quad (7d)$$

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Richardson and Davis (2001, p. 6.6) recommend a minimum K_4 value of 0.4. Furthermore, J. Sterling Jones (Federal Highway Administration, oral commun., 2002) recommends that measured scour be compared to calculated scour only if the measured approach velocity (V_o) exceeds $0.4V_{i50}$ because bed material theoretically would not be scoured at smaller approach velocities. Scour measurements made for approach velocities less than $0.4V_{i50}$ likely reflect scour holes that are remnants of an earlier flood. For this study, one measurement where $V_o < 0.4V_{i50}$ is included for comparison purposes (site 53, measurement 358, table 2).

When the approach velocity exceeds the critical velocity for the coarsest size fraction ($V_o > V_{ic95}$ or V_{ic90}), J. Sterling Jones (Federal Highway Administration, oral commun., 2002) recommends that K_4 default to 1.0. However, the HEC-18/Mueller equation for K_4 (equation 7) is based on data that include scour measurements where $V_o > V_{ic95}$ (David S. Mueller, U.S. Geological Survey, oral commun., 2002). Therefore, when comparing measured scour to calculated scour in this study, K_4 was calculated by equation 7, even when $V_o > V_{ic95}$. However, when designing or analyzing a bridge, K_4 values should typically be no lower than 1.0 when V_o exceeds V_{ic95} .

Table 4 summarizes the characteristics of the data used to develop equation 7 (Mueller, 1996, p. 93). The HEC-18/Mueller equation might not be reliable when applied outside the ranges of variables listed in table 4. Furthermore, according to the Federal Highway Administration (J. Sterling Jones, oral commun., 2002) certain D_{50}/D_{95} combinations can provide erratic results when using equation 7, and further refinement of the K_4 correction factor is being evaluated. The HEC-18/Mueller equation is included as a pier-scour calculation option within the computer model HEC-RAS, Version 3.1 (Brunner, 2002).

Evaluation of Pier-Scour Equations for Coarse-Bed Streams

Pier-scour depths calculated by the five equations were compared to 42 pier-scour measurements. In this section, comparisons for the results from each equation are discussed and presented using scatter and boxplots.

The equations were evaluated using two criteria, both of which are important to bridge design and to analysis of existing structures:

1. Number and magnitude of underestimates need to be minimal, and
2. Calculated scour depths need to match measured scour depths as closely as possible.

Results indicating the performance of the five equations are presented in table 5, and illustrated in figures 2 and 3. The scatterplots in figure 2 compare calculated pier scour for each of the five equations to measured pier scour for each measurement at each site. Generally, the most reliable equations result in calculated pier scour values that are mostly greater than mea-

sured values, but plot close to the line labeled “calculated pier scour equals measured pier scour.” Similarly, the boxplots of residuals (calculated scour minus measured scour) in figure 3 can be used to visually assess reliability of the five equations. On this basis, the most reliable equations are associated with boxes that are short, but that generally lie above the line indicating zero residual depth of scour.

Some error typically is involved in the measurement of pier scour; this error usually is estimated at the time the measurement is made. For the coarse-bed streams in this study, measurement errors ranged from 0.3 ft to 1.0 ft. In some instances, the measurement error was greater than the calculated residual (tables 2 and 5).

Simplified Chinese Equation

Pier-scour depths calculated using the Simplified Chinese equation were smaller than measured scour depths for 15 of the 18 measurements from Montana and for 14 of the 24 measurements from the 4 other States (table 5, figs. 2A and 3). However, 86 percent of the underestimated scour depths came within 2 ft of the measured scour. This equation occasionally produced pier-scour estimates that were less than zero. Calculated negative pier-scour values are reported in table 5 and plotted in figure 2A as zero. Calculated negative pier scour values also are assumed to be zero for purposes of calculating residuals reported in table 5 and shown in figure 3. Calculated scour depth was less than measured scour depth by about 2 to 5 ft at P4, site 1 in Alaska, where substantial debris was reported for measurements 7 and 8. Debris on the pier probably contributed to the measured scour and accounted for the underestimation by the equation. For the 13 measurements where calculated scour was greater than measured scour, the difference between calculated and measured scour ranged from 0.2 to 1.6 ft.

Froehlich Equation

Results from the Froehlich equation were similar to those from the Simplified Chinese equation (table 5, figs. 2A, 2B, and 3). The Froehlich equation is based on a regression analysis, where underestimates are as likely as overestimates. Pier-scour depths calculated using the Froehlich equation were smaller than measured scour depths for 14 of the 18 measurements from Montana and for 15 of the 24 measurements from the other States. Seventy-six percent of the underestimates were within 2 ft of measured scour.

Substantial debris on the pier at site 1 in Alaska likely explains why calculated scour depth was substantially less than measured scour depth for measurements 7 and 8. For the 12 measurements where calculated scour was greater than measured scour, the difference between measured and calculated scour ranged from 0.1 to 0.6 ft.

Table 4. Summary of scour measurement data used to develop the HEC-18/Mueller equation¹

[D_x is the particle size for which “x” percent of the bed material is finer. V_{c50} is the critical incipient motion velocity for particles of size D_{50} ; y_o , depth of flow directly upstream from the pier, in feet. Abbreviations: ft, foot; ft/s, foot per second; mm, millimeter; mi², square mile]

| Variable | Units | Number of measurements | Minimum | Median | Maximum | Mean | Standard deviation |
|------------------------------------|-----------------|------------------------|---------|--------|---------|-------|--------------------|
| Depth of scour, y_s | ft | 224 | 0 | 2.0 | 25.1 | 3.1 | 3.5 |
| Approach velocity, V_o | ft/s | 224 | .6 | 3.7 | 14.7 | 4.6 | 3.1 |
| Approach depth, y_o | ft | 224 | 0 | 13.4 | 39.2 | 14.5 | 8.9 |
| D_{50} | mm | 224 | .18 | .84 | 108 | 12 | 23 |
| D_{84} | mm | 220 | .37 | 3.3 | 233 | 27 | 43 |
| D_{95} | mm | 195 | .48 | 7.0 | 350 | 33 | 65 |
| Gradation coefficient ² | dimensionless | 220 | 1.3 | 2.3 | 12 | 3.5 | 2.5 |
| Pier width, b | ft | 224 | 1.2 | 3.4 | 15.1 | 4.4 | 2.8 |
| Drainage area | mi ² | 156 | 64 | 1,420 | 60,700 | 7,670 | 16,500 |
| Channel slope | dimensionless | 178 | .00016 | .00075 | .0050 | .0010 | .0011 |
| y_o/b | dimensionless | 224 | .12 | 3.4 | 14 | 4.0 | 2.9 |
| b/D_{50} | dimensionless | 224 | 8.5 | 990 | 14,200 | 1,890 | 3,180 |
| Froude number | dimensionless | 224 | .039 | .22 | .83 | .24 | .17 |
| V_o/V_{c50} | dimensionless | 224 | .44 | 1.1 | 5.4 | 1.3 | .9 |
| y_s/b | dimensionless | 224 | 0 | .6 | 2.1 | .7 | .4 |

¹Data from Mueller (1996).
²Gradation coefficient = D_{84}/D_{50}

The Froehlich equation calculated about 1 ft of scour for all the floods at the Montana sites, even though the measured scour varied from 0.3 to 5.5 ft. The calculated scour most likely did not vary because the Montana bridges have similar bed-material sizes and pier widths (table 2).

Froehlich Design Equation

Pier-scour depths calculated using the Froehlich design equation exceeded measured scour depths for every measurement except measurements 231 and 232, at site 32 in Montana (table 5, figs. 2C, and 3). Pier scour calculated using the Froehlich design equation for measurement 231 was only 0.6 ft less than measured pier scour, and for measurement 232, calculated scour was equal to measured scour. Pier-scour estimates obtained by this equation tend to be greatly influenced by the pier-width variable (fig. 2C).

At site 4 in Alaska, pier-scour depths calculated using the Froehlich design equation were more than 15 ft greater than measured scour depths. This large overestimation is likely due to the 15-ft-wide pier. Overestimates from the Froehlich design equation at all other sites ranged from 0.1 to 5.4 ft. Overestimates calculated using the HEC-18/Jones equation and the HEC-18/Mueller equation also were larger at site 4 than at all other sites.

HEC-18/Jones Equation

Pier-scour depths calculated using the HEC-18/Jones equation exceeded measured pier scour for all 42 observations (table 5, figs. 2D and 3). Many overestimates were larger for the HEC-18/Jones equation than for the Froehlich design equa-

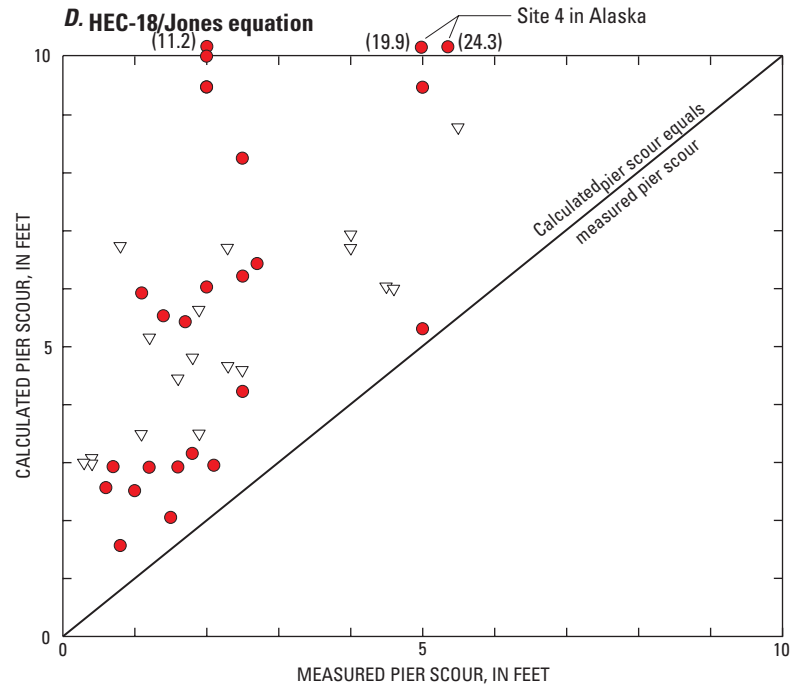
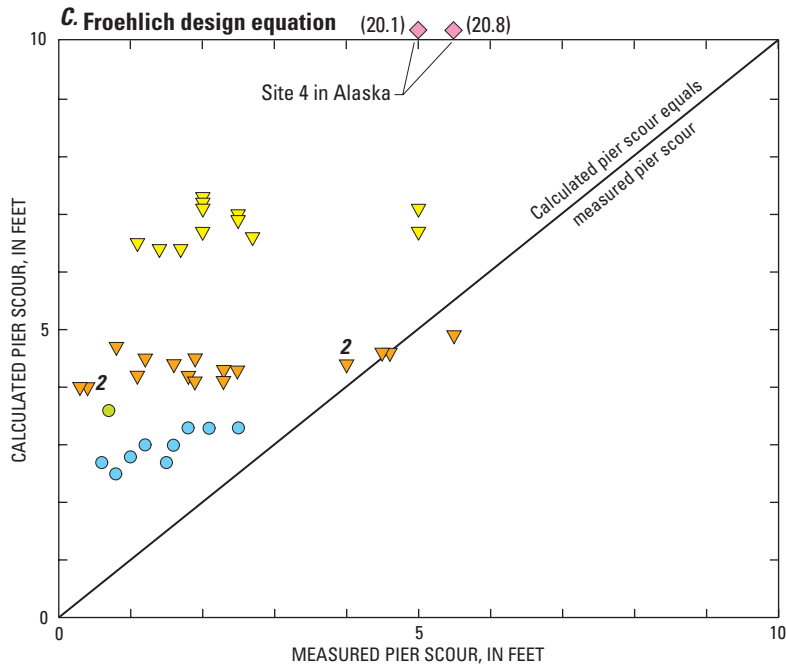
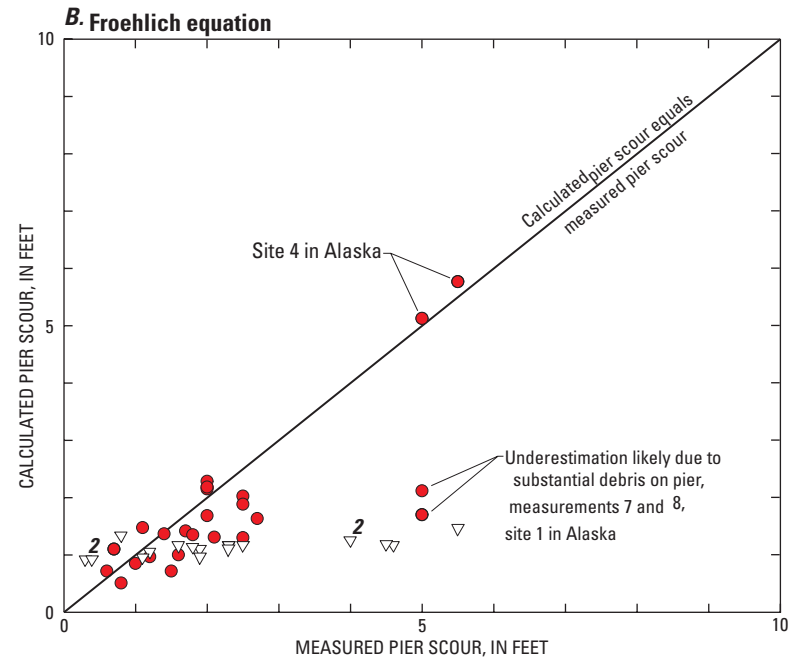
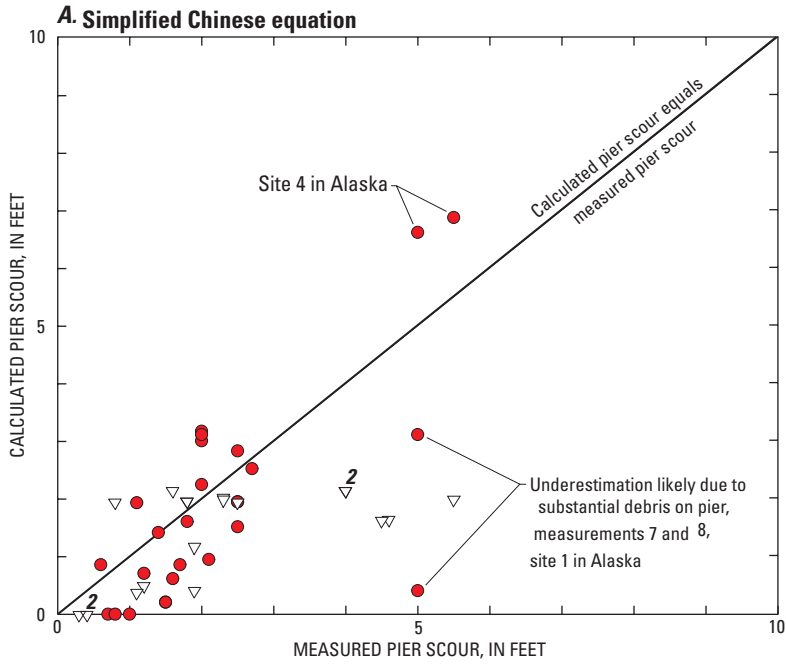
tion. The HEC-18/Jones equation overestimated pier scour at site 4 in Alaska by 14.9 and 18.8 ft (measurements 29 and 30, respectively, table 5). As discussed above, this large overestimation is likely due to the 15-ft-wide pier at this bridge. Overestimates at other sites ranged from 0.3 to 9.2 ft.

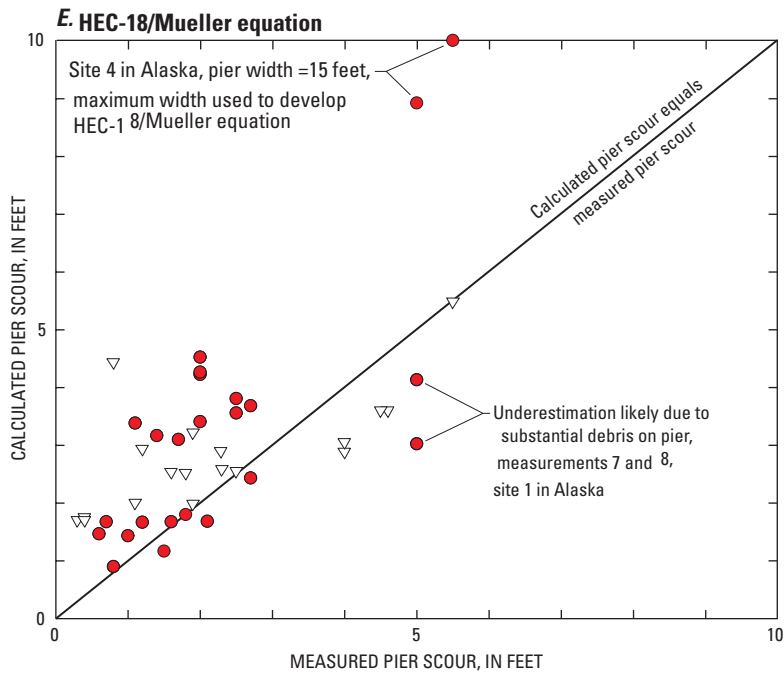
HEC-18/Mueller Equation

Pier-scour depths calculated using the HEC-18/Mueller equation were seldom less than measured pier-scour depths (table 5, figs. 2E and 3). Calculated scour depths were less than measured scour depths for 8 of the 42 observations, but 2 of these 8 underestimates were probably due to substantial debris on the pier at site 1 (measurements 7 and 8) in Alaska. For the other six observations where scour was underestimated, the difference between calculated and measured scour ranged from 0.1 to 1.1 ft.

The HEC-18/Mueller equation resulted in lower K_4 values than did the HEC-18/Jones equation, and the minimum allowable K_4 value (0.4) in the Mueller version is lower than the minimum K_4 value (0.7) recommended by the FHWA for the HEC-18/Jones equation. Therefore, the HEC-18/Mueller equation resulted in smaller calculated pier-scour depths than the HEC-18/Jones equation. Calculated scour depths from the HEC-18/Mueller equation were in better agreement with measured scour depths compared to calculated scour depths by the other equations that did not underestimate pier scour.

At site 4 in Alaska, where the pier width is 15 ft, the HEC-18/Mueller equation overestimated pier scour by 3.9 and 4.5 ft (tables 2 and 5). This disparity might be explained by the fact that 15 ft is the maximum pier width in the data set used to





EXPLANATION

For graphs A, B, D, E:

- 2 ▽ MONTANA
- OTHER STATES

For graph C:

PIER WIDTH, IN FEET

- 2.0, Virginia
- 2.5, Ohio
- 2 ▽ 3.0-3.4, Montana
- ▽ 5.0, Maryland
- ◇ 15, Alaska

(20.8) Numeral in parentheses indicates calculated pier scour greater than graph maximum

(*2*0.8) Numeral in parentheses indicates number of observations where scour measurements are equal and scour estimates are equal)

Figure 2. Comparison of calculated to measured pier scour for five equations. Data from U.S. Geological Survey Bridge Scour Data Management System (Chad R. Wagner, U.S. Geological Survey, written commun., 2002).

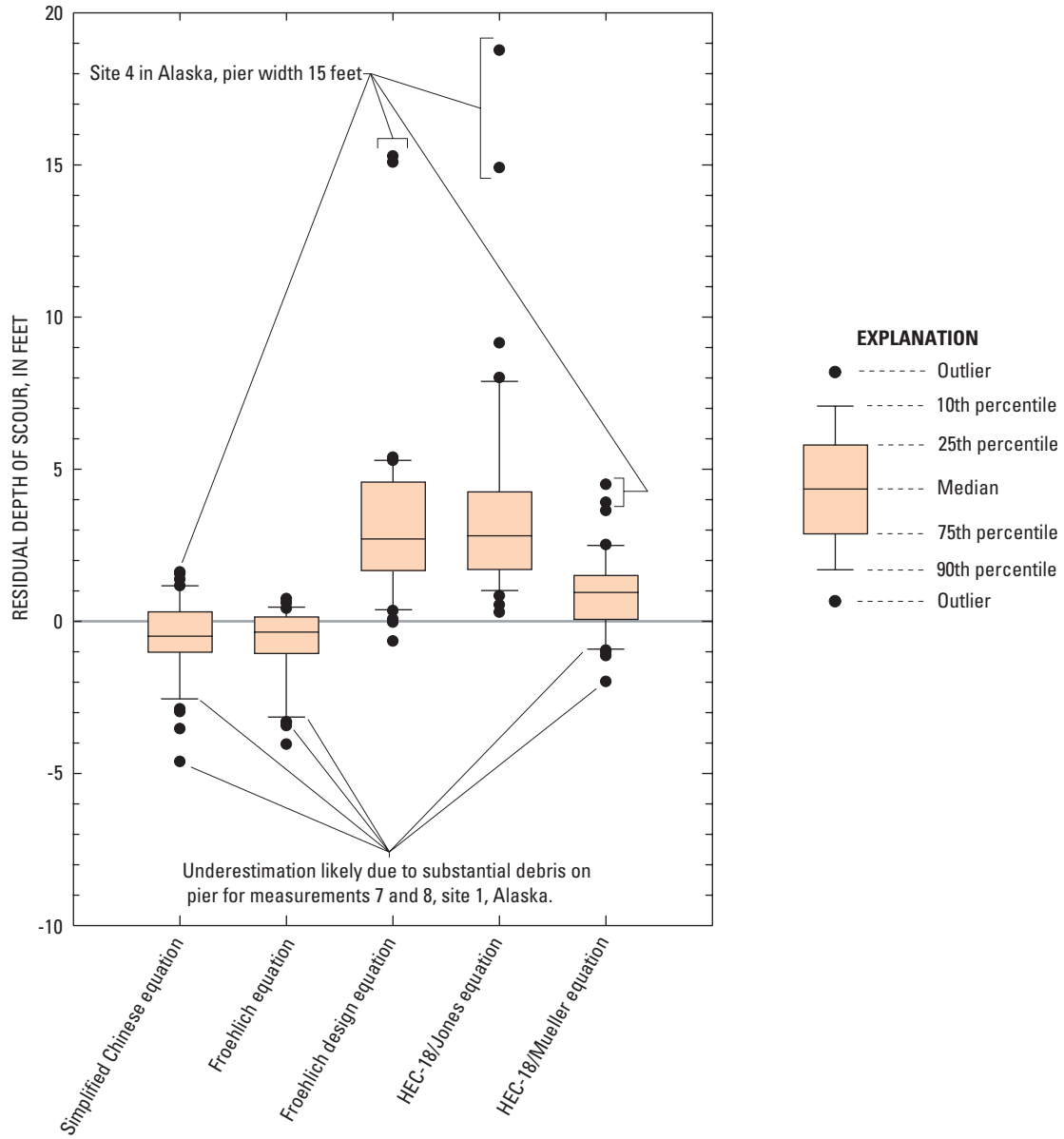


Figure 3. Distribution of residual depth of pier scour (calculated scour minus the measured scour). Residuals less than 0.0 indicate pier scour was underestimated.

Table 5. Comparison of calculated pier scour to measured pier scour from five equations

[All data for measured and calculated pier scour, measurement error, and residual are reported in feet. Pier identification: number or location of pier at which measurement took place. Residual: calculated scour minus measured scour, negative residual values indicate underestimated scour. Figures might not add to totals because of independent rounding. Abbreviations: BSDMS, Bridge Scour Data Management System. Symbol: --, not applicable]

| BSDMS site number | Measurement number ¹ | Date | Pier identification | Measured scour | Measurement error | Equation | | | | | | | | | |
|-------------------|---------------------------------|----------|---------------------|----------------|-------------------|--------------------|----------|------------------|----------|------------------|----------|------------------|----------|------------------|----------|
| | | | | | | Simplified Chinese | | Froehlich | | Froehlich design | | HEC-18/Jones | | HEC-18/Mueller | |
| | | | | | | Calculated scour | Residual | Calculated scour | Residual | Calculated scour | Residual | Calculated scour | Residual | Calculated scour | Residual |
| Montana | | | | | | | | | | | | | | | |
| 32 | 228 | 06/06/91 | P1 | 0.8 | 0.3 | 1.9 | 1.1 | 1.3 | 0.5 | 4.7 | 3.9 | 6.7 | 5.9 | 4.5 | 3.7 |
| 32 | 229 | 06/18/92 | P1 | 1.2 | .3 | .5 | -.7 | 1.1 | -.1 | 4.5 | 3.3 | 5.2 | 4.0 | 3.0 | 1.8 |
| 32 | 230 | 06/23/93 | P1 | 1.9 | .3 | 1.2 | -.7 | 1.1 | -.8 | 4.5 | 2.6 | 5.6 | 3.7 | 3.2 | 1.3 |
| 32 | 231 | 06/06/91 | P2 | 5.5 | .5 | 2.0 | -3.5 | 1.5 | -4 | 4.9 | -.6 | 8.8 | 3.3 | 5.5 | 0.0 |
| 32 | 232 | 06/18/92 | P2 | 4.6 | .5 | 1.6 | -3 | 1.2 | -3.4 | 4.6 | 0.0 | 6.0 | 1.4 | 3.6 | -1.0 |
| 32 | 233 | 06/23/93 | P2 | 4.5 | .5 | 1.6 | -2.9 | 1.2 | -3.3 | 4.6 | .1 | 6.0 | 1.5 | 3.6 | -.9 |
| 33 | 234 | 05/21/93 | P1 | 2.5 | .5 | 1.9 | -.6 | 1.2 | -1.3 | 4.3 | 1.8 | 4.6 | 2.1 | 2.6 | .1 |
| 33 | 235 | 05/27/93 | P1 | 2.3 | .5 | 2.0 | -.3 | 1.2 | -1.1 | 4.3 | 2.0 | 4.7 | 2.4 | 2.6 | .3 |
| 33 | 236 | 06/30/93 | P1 | 1.9 | .5 | .4 | -1.5 | 1.0 | -.9 | 4.1 | 2.2 | 3.5 | 1.6 | 2.0 | .1 |
| 33 | 237 | 05/21/93 | P2 | 1.6 | .3 | 2.1 | .5 | 1.2 | -.4 | 4.4 | 2.8 | 4.4 | 2.8 | 2.5 | .9 |
| 33 | 238 | 05/27/93 | P2 | 1.8 | .3 | 2.0 | .2 | 1.1 | -.7 | 4.2 | 2.4 | 4.8 | 3.0 | 2.5 | .7 |
| 33 | 239 | 06/30/93 | P2 | 1.1 | .3 | .4 | -.7 | 1.0 | -.1 | 4.2 | 3.1 | 3.5 | 2.4 | 2.0 | .9 |
| 33 | 240 | 05/21/93 | P3 | .3 | .3 | ² 0.0 | -.3 | .9 | .6 | 4.0 | 3.7 | 3.0 | 2.7 | 1.7 | 1.4 |
| 33 | 241 | 05/27/93 | P3 | .4 | .3 | ² 0.0 | -.4 | .9 | .5 | 4.0 | 3.6 | 3.1 | 2.7 | 1.8 | 1.4 |
| 33 | 242 | 06/30/93 | P3 | .4 | .3 | ² 0.0 | -.4 | .9 | .5 | 4.0 | 3.6 | 3.0 | 2.6 | 1.7 | 1.3 |
| 33 | -- | 06/12/96 | P1 | 4.0 | .5 | 2.1 | -1.9 | 1.3 | -2.7 | 4.4 | .4 | 6.9 | 2.9 | 3.1 | -.9 |
| 33 | -- | 06/09/97 | P1 | 4.0 | .5 | 2.2 | -1.8 | 1.3 | -2.7 | 4.4 | .4 | 6.7 | 2.7 | 2.9 | -1.1 |
| 85 | -- | 06/11/96 | P2 | 2.3 | .5 | 2.0 | -.3 | 1.1 | -1.2 | 4.1 | 1.8 | 6.7 | 4.4 | 2.9 | .6 |
| Alaska | | | | | | | | | | | | | | | |
| 1 | 1 | 07/02/71 | P1 | 2.5 | .5 | 1.5 | -1.0 | 2.0 | -.5 | 7.0 | 4.5 | 6.2 | 3.7 | 3.6 | 1.1 |
| 1 | 2 | 08/11/71 | P1 | 2.0 | .5 | 3.1 | 1.1 | 2.1 | .1 | 7.1 | 5.1 | 10.0 | 8.0 | 4.2 | 2.2 |
| 1 | 3 | 07/02/71 | P2 | 2.5 | .5 | 2.8 | .3 | 1.9 | -.6 | 6.9 | 4.4 | 8.3 | 5.8 | 3.8 | 1.3 |
| 1 | 4 | 08/11/71 | P2 | 2.0 | 1.0 | 3.2 | 1.2 | 2.3 | .3 | 7.3 | 5.3 | 9.5 | 7.5 | 4.3 | 2.3 |
| 1 | 5 | 07/02/71 | P3 | 2.0 | .5 | 2.2 | .2 | 1.7 | -.3 | 6.7 | 4.7 | 6.0 | 4.0 | 3.4 | 1.4 |
| 1 | 6 | 08/11/71 | P3 | 2.0 | 1.0 | 3.0 | 1.0 | 2.2 | .2 | 7.2 | 5.2 | 11.2 | 9.2 | 4.5 | 2.5 |
| 1 | 7 | 07/02/71 | P4 | 5.0 | .5 | .4 | -4.6 | 1.7 | -3.3 | 6.7 | 1.7 | 5.3 | .3 | 3.0 | -2.0 |
| 1 | 8 | 08/11/71 | P4 | 5.0 | 1.0 | 3.1 | -1.9 | 2.1 | -2.9 | 7.1 | 2.1 | 9.5 | 4.5 | 4.1 | -.9 |
| 4 | 29 | 09/02/71 | P1 | 5.0 | .5 | 6.6 | 1.6 | 5.1 | .1 | 20.1 | 15.1 | 19.9 | 14.9 | 8.9 | 3.9 |
| 4 | 30 | 09/04/71 | P1 | 5.5 | .5 | 6.9 | 1.4 | 5.8 | .3 | 20.8 | 15.3 | 24.3 | 18.8 | 10.0 | 4.5 |

Table 5. Comparison of calculated pier scour to measured pier scour from five equations—Continued

| BSDMS site number | Measurement number ¹ | Date | Pier identification | Measured scour | Measurement error | Equation | | | | | | | | | |
|-------------------|---------------------------------|----------|---------------------|----------------|-------------------|--------------------|----------|------------------|----------|------------------|----------|------------------|----------|------------------|----------|
| | | | | | | Simplified Chinese | | Froehlich | | Froehlich design | | HEC-18/Jones | | HEC-18/Mueller | |
| | | | | | | Calculated scour | Residual | Calculated scour | Residual | Calculated scour | Residual | Calculated scour | Residual | Calculated scour | Residual |
| Maryland | | | | | | | | | | | | | | | |
| 23 | 153 | 07/13/90 | Left | 1.1 | 1.0 | 1.9 | .8 | 1.5 | .4 | 6.5 | 5.4 | 5.9 | 4.8 | 3.4 | 2.3 |
| 23 | 154 | 04/01/93 | Left | 1.4 | 1.0 | 1.4 | 0.0 | 1.4 | 0.0 | 6.4 | 5.0 | 5.5 | 4.1 | 3.2 | 1.8 |
| 23 | 155 | 07/13/90 | Right | 2.7 | 1.0 | 2.5 | -2 | 1.6 | -1.1 | 6.6 | 3.9 | 6.4 | 3.8 | 3.7 | 1.0 |
| 23 | 156 | 04/01/93 | Right | 1.7 | 1.0 | .9 | -8 | 1.4 | -3 | 6.4 | 4.7 | 5.4 | 3.7 | 3.1 | 1.4 |
| Ohio | | | | | | | | | | | | | | | |
| 44 | 288 | 12/19/90 | P2 | .7 | .3 | ² 0.0 | -7 | 1.1 | .4 | 3.6 | 2.9 | 2.9 | 2.2 | 1.7 | 1.0 |
| Virginia | | | | | | | | | | | | | | | |
| 53 | 358 | 05/03/89 | P2 | .8 | 1.0 | ² 0.0 | -8 | .5 | -3 | 2.5 | 1.7 | 1.6 | .8 | .9 | .1 |
| 53 | 359 | 05/07/89 | P2 | .6 | 1.0 | .9 | .3 | .7 | .1 | 2.7 | 2.1 | 2.6 | 2.0 | 1.5 | .9 |
| 53 | 360 | 04/22/92 | P2 | 1.6 | 1.0 | .6 | -1.0 | 1.0 | -6 | 3.0 | 1.4 | 2.9 | 1.3 | 1.7 | .1 |
| 53 | 361 | 05/03/89 | P3 | 1.0 | 1.0 | 0.0 | -1.0 | .8 | -2 | 2.8 | 1.8 | 2.5 | 1.5 | 1.4 | .4 |
| 53 | 362 | 05/07/89 | P3 | 1.2 | 1.0 | .7 | -5 | 1.0 | -2 | 3.0 | 1.8 | 2.9 | 1.7 | 1.7 | .5 |
| 53 | 363 | 04/22/92 | P3 | 2.5 | 1.0 | 2.0 | -5 | 1.3 | -1.2 | 3.3 | .8 | 4.2 | 1.9 | 2.6 | .1 |
| 55 | 376 | 03/29/91 | P2 | 1.5 | 1.0 | .2 | -1.3 | .7 | -8 | 2.7 | 1.2 | 2.0 | .5 | 1.2 | -3 |
| 55 | 377 | 06/05/92 | P2 | 2.1 | 1.0 | 1.0 | -1.1 | 1.3 | -8 | 3.3 | 1.2 | 2.9 | .8 | 1.7 | -4 |
| 55 | 378 | 03/24/93 | P2 | 1.8 | 1.0 | 1.6 | -2 | 1.3 | -5 | 3.3 | 1.5 | 3.2 | 1.4 | 1.8 | 0.0 |

¹Measurement number from Landers and Mueller (1996).

²Simplified Chinese equation resulted in negative values for calculated pier scour; calculated scour is reported as zero.

develop the HEC-18/Mueller equation (table 4). As discussed above, the Froehlich design equation and the HEC-18/Jones equation overestimated pier scour at site 4 by 14.9 to 18.8 ft. Richardson and Davis (2001, p. 6.7) define a correction factor, K_w , that adjusts the HEC-18/Mueller equation for wide piers. However, the ratio of pier width to the D_{50} -particle size (b/D_{50}) at site 4 did not meet the criteria for using the K_w correction factor. In addition, Richardson and Davis recommend that K_w be used with caution because K_w is based only on few data from flume experiments. For these reasons, K_w was not applied to the calculations in this study.

Summary Statistics for Calculated and Measured Pier Scour

Statistics for calculated and measured pier scour are summarized in table 6. The averages of pier-scour depths calculated from all five equations ranged from 1.5 to 6.1 ft; the average of measured pier-scour depths was 2.2 ft. Average calculated pier-scour depths from the Froehlich equation (1.5 ft), Simplified Chinese equation (1.7 ft), and the HEC-18/Mueller equation (3.1 ft) were relatively close to the average of measured pier-scour depths (2.2 ft). Although averages of calculated pier-scour depths from the Froehlich and Simplified Chinese equations were closer to the average of measured pier scour depths, these equations resulted in the most underestimates of all the equations.

The HEC-18/Mueller equation resulted in relatively few underestimates, with an average underestimate of 0.9 ft. For measurements where the HEC-18/Mueller equation overestimated scour, the average overestimate was 1.4 ft. This average overestimate was smaller than overestimates from the other equations that did not underestimate scour (Froehlich design and HEC-18/Jones equations).

Summary and Conclusions

Five pier-scour equations were evaluated based on existing pier-scour, bed-material, bridge, and streamflow data for coarse-bed streams in Montana, Alaska, Maryland, Ohio, and Virginia. Pier scour-depths calculated for flood events with approximate recurrence intervals of less than 2 to 100 years by the Simplified Chinese equation, the Froehlich equation, the Froehlich design equation, the HEC-18/Jones equation, and the HEC-18/Mueller equation were compared to 18 pier-scour measurements at 3 bridge sites in Montana and 24 pier-scour measurements at 6 bridge sites in 4 other states. Site information and measurements used in this study are summarized in the report.

When applied to data from the bridge sites, the equations produced results that were consistent between Montana and the four other states. The Simplified Chinese and Froehlich

Table 6. Summary statistics for calculated pier scour for five pier-scour equations and measured pier scour

[Statistics include data for all 42 sites. Symbol: --, not applicable]

| Method | Average calculated scour ¹ (feet) | Standard deviation (feet) | Average ² under-estimation (feet) | Number of under-estimations | Average ³ over-estimation (feet) | Number of over-estimations |
|---------------------------------|--|---------------------------|--|-----------------------------|---|----------------------------|
| Simplified Chinese ⁴ | 1.7 | 1.5 | 1.2 | 28 | 0.8 | 13 |
| Froehlich ⁴ | 1.5 | 1.0 | 1.2 | 29 | .3 | 12 |
| Froehlich design ⁴ | 5.6 | 3.7 | .6 | 1 | 3.5 | 40 |
| HEC-18/Jones | 6.1 | 4.4 | -- | 0 | 3.9 | 42 |
| HEC-18/Mueller ⁵ | 3.1 | 1.8 | .9 | 8 | 1.4 | 32 |
| Measured | 2.2 | 1.5 | -- | -- | -- | -- |

¹Data from table 5.

²Absolute value of average of differences between scour calculated using indicated equation and measured scour for measurements where scour was underestimated.

³Average of differences between scour calculated using indicated equation and measured scour for measurements where scour was overestimated.

⁴The Simplified Chinese, Froehlich, and Froehlich design equations, each accurately estimated scour for one measurement (residual values were equal to 0.0, table 5).

⁵The HEC-18/Mueller equation accurately estimated pier scour for two measurements (residual values were equal to 0.0, table 5).

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equations both underestimated scour at several sites. Eighty-six percent of the underestimates from the Simplified Chinese equation were less than 2 ft, and 76 percent of the underestimates from the Froehlich equation were less than 2 ft. The HEC-18/Jones equation resulted in the largest overestimates for the equations examined in the study. The Froehlich design equation also overestimated scour, although overestimates were smaller than for the HEC-18/Jones equation.

The HEC-18/Mueller equation generally predicted scour better than the other four equations according to the evaluation criteria used in this study. Scour was seldom underestimated, and calculated scour was closer to measured scour than for the other equations that did not underestimate scour. This equation might not be reliable when applied to sites where variables are outside the ranges within the dataset from which the equation was derived. This equation is being revised and sometimes can produce erratic results for certain combinations of D_{50}/D_{95} . Measurements of pier scour at additional sites are needed to further evaluate the HEC-18/Mueller equation. Additional scour measurements during rarer events, such as the 100-year and 500-year flood, also are needed to further test the equation.

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